. . . more tests . . .

1. Use the comparison test to determine whether or not the series converges.

(a)

$$\sum_{k=1}^{\infty} \frac{1}{3^k+2}$$

Note the following:

$$\frac{1}{3^k+2} < \frac{1}{3^k} \text{ for all } k \geq 0 \Longrightarrow \sum_{k=1}^{\infty} \frac{1}{3^k+2} \leq \sum_{k=1}^{\infty} \frac{1}{3^k}$$

$$\sum_{k=1}^{\infty} \frac{1}{3^k} \text{is a convergent geometric series, with } r = \frac{1}{3} < 1$$

Therefore $\sum_{k=1}^{\infty} \frac{1}{3^k+2}$ converges by the Comparison Test.

(b)

$$\sum_{k=1}^{\infty} \frac{1+\ln k}{k}$$

Note the following:

$$\frac{1+\ln k}{k} > \frac{1}{k} \text{ for all } k \geq 1 \Longrightarrow \sum_{k=1}^{\infty} \frac{1+\ln k}{k} \geq \sum_{k=1}^{\infty} \frac{1}{k}$$

$$\sum_{k=1}^{\infty} \frac{1}{k} \text{ is a divergent p-series, } p = 1$$

Therefore $\sum_{k=1}^{\infty} \frac{1+\ln k}{k}$ diverges by the Comparison Test.

2. Use the limit comparison test to determine whether or not the series converges.

(a)

$$\sum_{k=1}^{\infty} \frac{k+2}{k^3-k+100}$$

Apply Limit Comparison Test, using $\sum \frac{1}{k^2}$:

$$\lim_{k\to+\infty} \frac{\frac{k+2}{k^3-k+100}}{\frac{1}{k^2}} = \lim_{k\to+\infty} \left(\frac{k+2}{k^3-k+100}\right)\left(\frac{k^2}{1}\right) = \lim_{k\to+\infty} \frac{k^3+2k^2}{k^3-k+100}$$

$$\lim_{k\to+\infty} \frac{3k^2+4k}{3k^2-1} = \lim_{k\to+\infty} \frac{6k+4}{6k} = \lim_{k\to+\infty} \frac{6}{6} = 1,\ 0 < 1 < +\infty$$

$\sum_1^{+\infty} \frac{1}{k^2}$ is a convergent p-series, $p = 2 > 1$. Therefore $\sum_{k=1}^{\infty} \frac{k+2}{k^3-k+100}$ converges by the Limit Comparison Test.

(b)

$$\sum_{k=1}^{\infty} \frac{k(k+3)}{(k+1)(k+2)(k+5)}$$

Apply Limit Comparison Test, using $\sum \frac{1}{k}$:

$$\lim_{k\to+\infty} \frac{\frac{k(k+3)}{(k+1)(k+2)(k+5)}}{\frac{1}{k}} = \lim_{k\to+\infty} \frac{k^3+3k^2}{k^3+8k^2+17k+10} = \cdots = 1,\ 0 < 1 < +\infty$$

$\sum_1^{+\infty} \frac{1}{k}$ is a divergent p-series, $p = 1$. Therefore $\sum_{k=1}^{+\infty} \frac{k(k+3)}{(k+1)(k+2)(k+5)}$ diverges by the Limit Comparison Test.