. . . last tests . . .

1. Use the ratio test to determine whether or not the series converges. If the test is inconclusive, then say so.

(a)

$$\sum_{k=1}^{\infty} \frac{3^k}{k^2}$$

$$\lim_{k\to+\infty} \frac{\frac{3^{k+1}}{(k+1)^2}}{\frac{3^k}{k^2}} = \lim_{k\to+\infty} \frac{3^k \cdot 3}{k^2+2k+1} \cdot \frac{k^2}{3^k} = \lim_{k\to+\infty} \frac{3k^2}{k^2+2k+1} =$$

$$\lim_{k\to+\infty} \frac{6k}{2k+2} = \lim_{k\to+\infty} \frac{6}{2} = 3 > 1$$

Therefore, the seres $\sum_{k=1}^{\infty} \frac{3^k}{k^2}$ diverges by the Ratio Test.

(b)

$$\sum_{k=1}^{\infty} \frac{k+10}{k^2+2}$$

$$\lim_{k\to+\infty} \frac{\frac{(k+1)+10}{(k+1)^2+2}}{\frac{k+10}{k^2+2}} = \lim_{k\to+\infty} \frac{k+11}{k^2+2k+3} \cdot \frac{k^2+2}{k+10} =$$

$$\lim_{k\to+\infty} \frac{k^3+11k^2+2k+22}{k^3+12k^2+23k+30} = \lim_{k\to+\infty} \frac{3k^2+22k+2}{3k^2+24k+23} =$$

$$\lim_{k\to+\infty} \frac{6k+22}{6k+24} = \lim_{k\to+\infty} \frac{6}{6} = 1$$

The Ratio Test is inconclusive. Note - the Ratio Test will be inconclusive for any series where the sequence of underlying terms looks like a rational function. Limit Comparison Test with the appropriate p-series will crack the convergence / divergence questions . . . .

2. Use the root test to determine whether or not the series converges. If the test is inconclusive, then say so.

(a)

$$\sum_{k=1}^{\infty} \left(\frac{k}{2500}\right)^k$$

$$\lim_{k\to+\infty} \sqrt[k]{\left(\frac{k}{2500}\right)^k} = \lim_{k\to+\infty} \frac{k}{2500} = +\infty > 1$$

The series $\sum_{k=1}^{\infty} \left(\frac{k}{2500}\right)^k$ diverges by the Root Test. Note that you could also use the Divergence Test.

(b)

$$\sum_{k=1}^{\infty} \frac{k}{3^k}$$

$$\lim_{k\to+\infty} \sqrt[k]{\frac{k}{3^k}} = \lim_{k\to+\infty} \frac{\sqrt[k]{k}}{3} = \frac{1}{3} < 1$$

The series $\sum_{k=1}^{\infty} \frac{k}{3^k}$ converges by the Root Test.