mono a mono

1. Use the recommended method to show that the given sequence is eventually strictly increasing or eventually strictly decreasing. Determine whether or not the sequence converges. If the sequence converges, find the limit.

(a) difference

$$\left\{3 - \frac{1}{n}\right\}_{n=1}^{+\infty}$$

$$a_{n+1} - a_n = \left(3 - \frac{1}{n+1}\right) - \left(3 - \frac{1}{n}\right) = \frac{1}{n(n+1)} > 0$$

$$\implies \text{sequence is strictly increasing}$$

(b) ratio

$$\left\{\frac{n!}{e^{2n}}\right\}_{n=1}^{+\infty}$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{e^{2(n+1)}}{(n+1)!}}{\frac{e^{2n}}{n!}} = \left(\frac{e^{2n}e^2}{(n+1)n!}\right)\left(\frac{n!}{e^{2n}}\right) = \frac{e^2}{n+1} < 1 \text{ for all } n > 7$$

$$\implies \text{the sequence is eventually strictly decreasing}$$

(c) differentiation

$$\left\{\tan^{-1} n\right\}_{n=1}^{+\infty}$$

$$\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2} > 0 \text{ for all } x$$

$\implies$ The function $f(x) = \tan^{-1} x$ is strictly increasing, so the related sequence is strictly increasing.

2. Suppose that $\{a_n\}$ is a monotone sequence such that $-1 \leq a_n \leq 1$. Must the sequence converge? If so, what can you say about the limit?

Answer: Yes, the sequence converges to some value in $[-1, 1]$.

3. Suppose that $\{b_n\}$ is a monotone sequence such that $b_n \leq 5$. Must the sequence converge? If so, what can you say about the limit?

No, it doesn't have to converge. Example: $\{-n\}_1^{\infty}$