just checking . . . .

1. Converge or diverge?

$$\sum_{n=1}^{+\infty} \frac{\tan^{-1} n}{n^{1.2}}$$

Consider $\sum \frac{1}{n^{1.2}}$ - this is a convergent p-series, $p = 1.2$.

$$\lim_{n\to+\infty} \frac{\frac{\tan^{-1} n}{n^{1.0}}}{\frac{1}{n^{1.2}}} = \lim_{n\to+\infty} \frac{n^{1.2}\tan^{-1} n}{n^{1.2}} = \lim_{n\to+\infty} \tan^{-1} n = \frac{\pi}{2}$$

So $\sum \frac{\tan^{-1} n}{n^{1.2}}$ converges by the Limit Comparison Test.

2. Converge or diverge?

$$1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \cdots = \sum_{n=0}^{+\infty} \frac{1}{2n+1}$$

Consider $\sum \frac{1}{n}$ - this is a divergent p-series, $p = 1$.

$$\lim_{n\to+\infty} \frac{\frac{1}{2n+1}}{\frac{1}{n}} = \lim_{n\to+\infty} \frac{n}{2n+1} = \lim_{n\to+\infty} \frac{1}{2} = \frac{1}{2}$$

So $\sum \frac{1}{2n+1}$ diverges by the Limit Comparison Test.

3. If the series converges, find its value.

$$\sum_{n=1}^{+\infty} \left[e^{\frac{1}{n}} - e^{\frac{1}{n+1}}\right]$$

$$a_1 = e - e^{\frac{1}{2}}, a_2 = e^{\frac{1}{2}} - e^{\frac{1}{3}}, a_3 = e^{\frac{1}{3}} - e^{\frac{1}{4}}, \ldots$$

$$s_1 = a_1 = e - e^{\frac{1}{2}}$$

$$s_2 = s_1 + a_2 = e - e^{\frac{1}{2}} + e^{\frac{1}{2}} - e^{\frac{1}{3}} = e - e^{\frac{1}{3}}$$

$$s_3 = s_2 + a_3 = e - e^{\frac{1}{3}} + e^{\frac{1}{3}} - e^{\frac{1}{4}} = e - e^{\frac{1}{4}}$$

$$\Longrightarrow s_n = e - e^{\frac{1}{n+1}}$$

Answer: $\displaystyle\sum_{n=1}^{\infty} \left[e^{\frac{1}{n}} - e^{\frac{1}{n+1}}\right] = \lim_{n\to+\infty} s_n = \lim_{n\to+\infty} \left[e - e^{\frac{1}{n+1}}\right] = e - 1$

4. Converge or diverge?

$$\sum_{n=1}^{+\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$$

$$\lim_{n\to+\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right) = \ln\left(\lim_{n\to+\infty} \frac{n^2+1}{2n^2+1}\right) = \ln\left(\frac{1}{2}\right) \neq 0$$

The series diverges by the Divergence Test.

5. Converge or diverge?

$$\sum_{n=1}^{+\infty} \frac{5^n}{(2n)!}$$

Apply ratio test (factorials)

$$\frac{a_{n+1}}{a_n} = \frac{\frac{5^{n+1}}{(2(n+1))!}}{\frac{5^n}{(2n)!}} = \frac{5^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{5^n} = \frac{5^n \cdot 5 \cdot (2n)!}{5^n \cdot (2n+2)(2n+1)(2n)!}$$

$$\lim_{n\to+\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to+\infty} \frac{5}{(2n+2)(2n+1)} = 0 < 1$$

The series converges by the Ratio Test.

6. Converge or diverge?

$$\sum_{n=1}^{+\infty} \tan\left(\frac{1}{n}\right)$$

Consider $\sum \frac{1}{n}$, a divergent p-series, $p = 1$.

$$\lim_{n\to+\infty} \frac{\tan\left(\frac{1}{n}\right)}{\frac{1}{n}} = \lim_{n\to+\infty} \frac{-\frac{1}{n^2}\sec^2\left(\frac{1}{n}\right)}{-\frac{1}{n^2}} = \lim_{n\to+\infty} \sec^2\left(\frac{1}{n}\right) = 1$$

The series $\sum \tan(1/n)$ diverges by Limit Comparison Test.