just checking . . . .

1. Converge or diverge? (Section 11.4 #13)

$$\sum_{n=1}^{+\infty} \frac{\tan^{-1} n}{n^{1.2}}$$

$$\lim_{n\to+\infty} \frac{\frac{\tan^{-1} n}{n^{1.2}}}{\frac{1}{n^{1.2}}} = \lim_{n\to+\infty} \frac{n^{1.2}\tan^{-1} n}{n^{1.2}} = \lim_{n\to+\infty} \tan^{-1} n = \frac{\pi}{2}$$

$\sum \frac{1}{n^{1.2}}$ is a convergent p-series, $p = 1.2 > 1$. So $\sum \frac{\tan^{-1} n}{n^{1.2}}$ converges by the Limit Comparison Test.

2. Converge or diverge? (Sectio 11.3 #13)

$$1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \cdots = \sum_{n=0}^{+\infty} \frac{1}{2n+1}$$

$$\lim_{n\to+\infty} \frac{\frac{1}{2n+1}}{\frac{1}{n}} = \lim_{n\to+\infty} \frac{n}{2n+1} = \lim_{n\to+\infty} \frac{1}{2} = \frac{1}{2}$$

$\sum \frac{1}{n}$ is a divergent p-series, $p = 1$. So $\sum \frac{1}{2n+1}$ diverges by the Limit Comparison Test.

3. If the series converges, find its value. (Section 11.2 #39)

$$\sum_{n=1}^{+\infty}\left[e^{\frac{1}{n}}-e^{\frac{1}{n+1}}\right]$$

Underlying sequence of terms and sequence of partial sums:

$$\begin{aligned}
a_1 &= e^{\frac{1}{1}}-e^{\frac{1}{2}} &\implies s_1 &= e^{\frac{1}{1}}-e^{\frac{1}{2}}\\
a_2 &= e^{\frac{1}{2}}-e^{\frac{1}{3}} &\implies s_2 &= e-e^{\frac{1}{3}}\\
a_3 &= e^{\frac{1}{3}}-e^{\frac{1}{4}} &\implies s_3 &= e-e^{\frac{1}{4}}\\
&\vdots
\end{aligned}$$

$$s_n = e - e^{\frac{1}{n+1}}$$

So . . .

$$\sum_{n=1}^{+\infty}\left[e^{\frac{1}{n}}-e^{\frac{1}{n+1}}\right] = \lim_{n\to+\infty} s_n = \lim_{n\to+\infty} e - e^{\frac{1}{n+1}} = e-1$$

The series converges to $e-1$.

4. Converge or diverge? (Section 11.2 #29)

$$\sum_{n=1}^{+\infty}\ln\left(\frac{n^2+1}{2n^2+1}\right)$$

$$\lim_{n\to+\infty}\ln\left(\frac{n^2+1}{2n^2+1}\right) = \ln\left(\lim_{n\to+\infty}\frac{n^2+1}{2n^2+1}\right) = \ln\left(\lim_{n\to+\infty}\frac{2n}{4n}\right) = \ln\frac{1}{2} \neq 0$$

$\sum \ln\left(\frac{n^2+1}{2n^2+1}\right)$ diverges by the Divergence Test.