Friday - Week 7

Warm-up

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2} = \frac{\ln(1)}{1} + \frac{\ln(2)}{4} + \frac{\ln(3)}{9} + \dots$$

Comparison / Limit Comparison Test:

Idea #1 $\qquad \frac{\ln(n)}{n^2} > \frac{1}{n^2} \qquad n \geq 1$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges. (it's the lower series $\Rightarrow$ Inconclusive)

L.C.T. $\Rightarrow$ $\frac{\frac{\ln(n)}{n^2}}{\frac{1}{n^2}} = \ln(n) \xrightarrow{\text{as } n \to \infty} \infty$

$\Rightarrow$ Inconclusive

L.C.T.

given: a_n

comparable b_n

$\frac{a_n}{b_n} \xrightarrow{=} L$ $\qquad \Sigma a_n, \Sigma b_n$ same

$a_n > L b_n$ $\quad \downarrow >$ $\quad \infty$: if Σb_n diverges then Σa_n diverges

$<$ $\quad 0$ $\quad$ $a_n < L b_n$ $\Rightarrow$ if Σb_n converges then Σa_n converges

Idea #2 $\qquad \frac{\ln(n)}{n^2} < \frac{n}{n^2} = \frac{1}{n^1}$

$\Sigma \frac{1}{n}$ diverges $\Rightarrow$ Inconclusive

Idea #3: Comparable: $\frac{1}{n^{3/2}}$

L.C.T.

$$\frac{\frac{\ln(n)}{n^2}}{\frac{1}{n^{3/2}}} = \frac{n^{3/2}}{1} \cdot \frac{\ln(n)}{n^2} = \frac{\ln(n)}{n^{1/2}} \xrightarrow{n \to \infty} \frac{\ln(\infty)}{\infty^{1/2}} = \frac{\infty}{\infty}$$

$\frac{3}{2} = 1.5$

$$= \lim_{n \to \infty} \frac{\frac{1}{n}}{\frac{1}{2} n^{-1/2}} = \lim_{n \to \infty} 2 \cdot \frac{1}{n} \cdot n^{1/2} = 2 \lim_{n \to \infty} \frac{1}{n^{1/2}} = 0$$

Does $\Sigma \frac{1}{n^{3/2}}$ converge?

yes — p-series

$p = \frac{3}{2} > 1$.

$\Rightarrow$ L.C.T. implies $\Sigma \frac{\ln(n)}{n^2}$ converges

$$\frac{\ln(n)}{n^2} \leq \frac{n}{n^2} = \frac{1}{n} \xrightarrow{} \text{div}$$

Comparison Test

$$\sum \frac{\ln(n)}{n^2}$$

$$\frac{\ln(n)}{n^2} < \frac{1}{n^{3/2}}$$

why?

Determine conv. / div

$= \frac{1}{n^2}$

① $\sum_{n=1}^{\infty} \frac{n^2+1}{n^4+2n}$

L.C.T.

$\frac{\frac{n^2+1}{n^4+2n}}{\frac{1}{n^2}} = \frac{n^4+n^2}{n^4+2n} \xrightarrow{\text{as } n\to\infty} 1 \neq 0 \quad \frac{\infty}{\infty}$

$\frac{n^2+1}{n^4+2n} \boxed{?} \frac{n^2}{n^4} = \frac{1}{n^2}$

$<$

$>$

Series behave the same

$\sum \frac{1}{n^2}$ converges $\Rightarrow$ given converges

② $\sum_{n=1}^{\infty} \frac{n^3+1}{n^4+n^2}$

LCT $\frac{\text{given}}{\frac{1}{n}} \longrightarrow 1$

$\sum \frac{1}{n}$ div $\Rightarrow$ given div.

Ratio Test:
(measures the rate of convergence)

Let $\sum a_n$ w/ non-zero terms

let $\rho = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$.

(Idea: $\sum \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{25}$

$\frac{1}{4}, \frac{\frac{1}{9}}{\frac{1}{4}}, \frac{\frac{1}{25}}{\frac{1}{9}}, \ldots$

1. $0 \leq \rho < 1 \Rightarrow \sum a_n$ converge

2. $\rho > 1 \Rightarrow \sum a_n$ diverge

3. $\rho = 1 \Rightarrow$???

Note: useful for factorials & n-exponents

Not this

$$\sum \frac{1}{n^2} \longrightarrow \frac{\frac{1}{(n+1)^2}}{\frac{1}{n^2}} = \frac{n^2}{(n+1)^2} \longrightarrow 1 \Rightarrow ???$$

Ex. $\sum \frac{2^n}{n!}$

$$\frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} = \frac{n!}{2^n} \cdot \frac{2^{n+1}}{(n+1)!} = \frac{\cancel{n!}\, \cancel{2^n} \cdot 2}{\cancel{2^n}\, \cancel{(n+1)!}}$$

$$= \frac{2}{n+1} \longrightarrow 0$$

$$\frac{n!}{(n+1)!} = \frac{n \cdot (n-1) \cdot (n-2) \cdots}{(n+1)\, n \cdot (n-1)\, (n-2)}$$

$$\frac{n!}{(n+1)!} = \frac{\cancel{n!}}{(n+1) \cdot \cancel{n!}}$$

Ratio Test
$\Rightarrow$ given series converge

exam: Next
- Wed/Thur

(Fri) — Rest Day

- study guide
- practice
- WeBWork