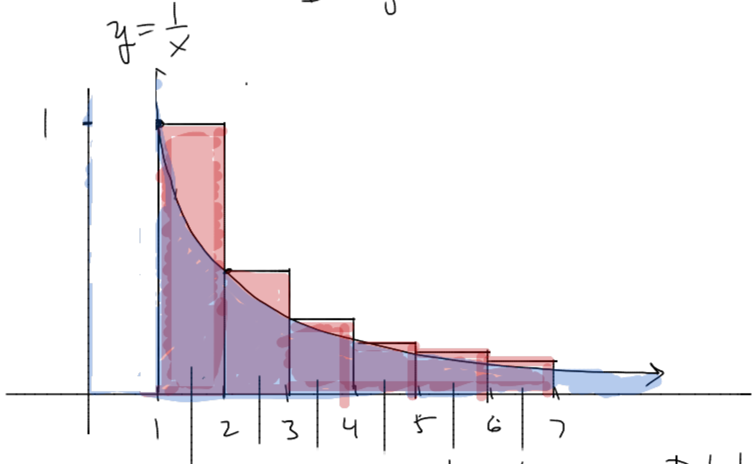
Integral Test

Fact:

Calc I		Calc II
Definite Integrals	$\xleftrightarrow{\text{related}}$	Infinite Series

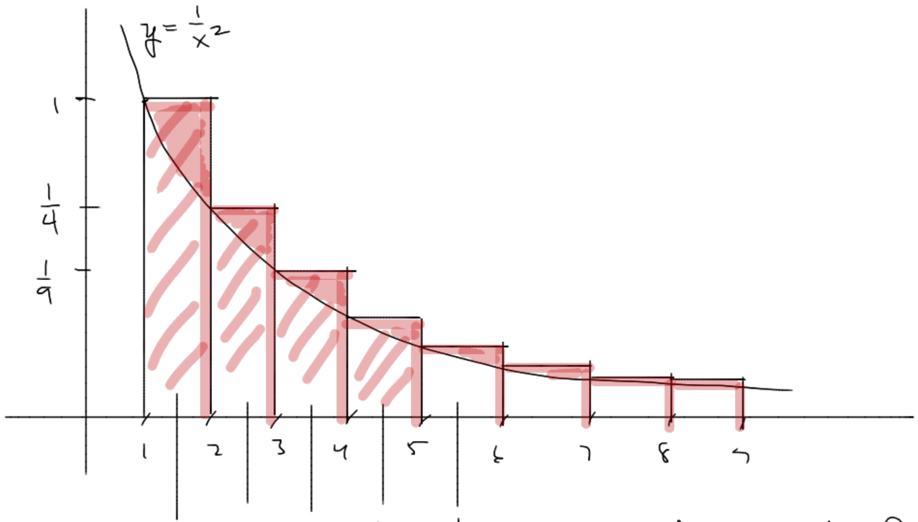


Area $1 \quad \frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6} \quad \ldots$

Total Area in "Red" Rectangles $= \sum_{n=1}^{\infty} \frac{1}{n} \geq \int_1^{\infty} \frac{1}{x}\,dx = \infty$

$\Longrightarrow$ thus $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges to infinity

Next —— Repeat:



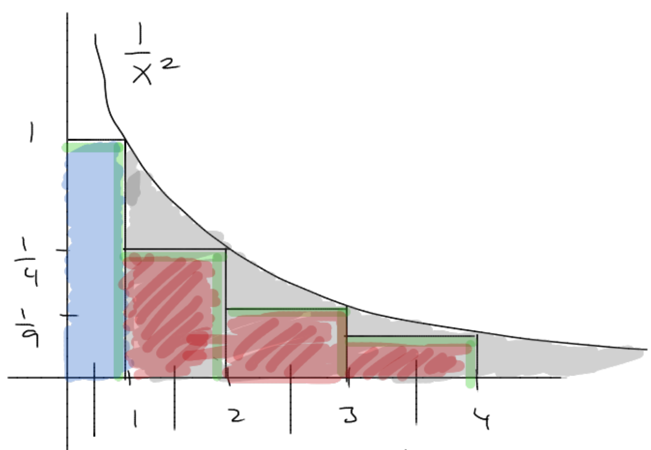
x^{-2}

Area $\quad 1 \quad \frac{1}{4} \quad \frac{1}{9} \quad \frac{1}{16} \quad \frac{1}{25} + \dots$ = Area of Rectangles $\geq \int_1^\infty \frac{1}{x^2}\,dx$

$$= \left.\frac{-1}{x}\right|_1^\infty = -\frac{1}{\infty} - \left(\frac{-1}{1}\right) = 0 + 1 = 1$$

$$\boxed{\sum_{n=1}^\infty \frac{1}{n^2} \geq 1}$$

this isn't saying much



Area: $1 \quad \frac{1}{4} \quad \frac{1}{9} \quad \frac{1}{16} \quad \Longrightarrow$

Total Area in Rectangles $\sum_{n=1}^\infty \frac{1}{n^2} = 1 +$ Red Rectangle $\leq 1 + \int_1^\infty \frac{1}{x^2}\,dx \leq 2$

$= 1$ (see above)

$\Rightarrow$ this series is convergent

(idea: b/c $\int_1^\infty \frac{1}{x^2}\,dx < \infty$ the sum must be $< \infty$ too.

Integral Test

<u>theorem</u>: Let $a_n = f(n)$ where $f(x)$ cts, decreasing, positive for $x \geq 1$

(1) If $\int_1^\infty f(x)\,dx$ converges then $\sum_{n=1}^{\infty} a_n$ converges

(2) If $\int_1^\infty f(x)\,dx$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges

<u>Idea</u>: If f is suitable — we integrate to determine conv / div ______

Ex Determine whether $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges or diverges

$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \cdots$

Since the integral diverges the series diverges too

Use the integral test ——— !

$$\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx = \int_1^{\infty} x^{-1/2}\,dx = 2x^{\frac{1}{2}}\Big|_1^{\infty} = 2\sqrt{\infty} - 2 = \infty$$

$\lim_{b\to\infty} 2\sqrt{x}\Big|_1^b = 2(\sqrt{b}-1)$
$= \infty$

this works b/c $f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

is
- cts
- decreasing when $x \geq 1$
- positive

decreasing: $f'(x) = -\frac{1}{2}x^{-\frac{3}{2}} = \frac{-1}{2\sqrt{x^3}}$

note: this is < 0 if $x \geq 1$

Wednesday:

—— p-Test! ——

$\frac{1}{n^{1/2}}$ here $p = \frac{1}{2}$

we just learned $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ " diverges

p-test will classify these results

$\sum_{n=1}^{\infty} \frac{1}{n}$	$\sum_{n=1}^{\infty} \frac{1}{n^2}$
$p = 1$	$p = 2$
$\downarrow$	$\downarrow$
∞	converges