Math 163 - Calculus - Exam 2 - Guide **Name:** ____________________

February 26, 2025

Show your work to receive full credit.

1.(1.1) Indicate absolute convergence, conditional convergence or divergence

$$\sum_{k=2}^{\infty} \frac{5k\sqrt{k}}{7k^3 + 5k + 1}$$

For large k : $\sum \frac{5k\sqrt{k}}{7k^3} = \sum \frac{5k^{3/2}}{7k^3} = \sum \frac{5}{7k^{3/2}}$ converges p - test $p = 3/2 > 1$

$$\frac{5k\sqrt{k}}{7k^3 + 5k + 1} < \frac{5}{7k^{3/2}} \quad \forall k$$

since the given series is bounded above by a convergent series, it must converge too

(1.2) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt[3]{2k+11}}$

$(-1)^k \Rightarrow$ ALT.

A.S.T. : 1. alt
2. terms decrease ... ours clearly decrease $\frac{1}{\sqrt[3]{2k+11}}$ ← const ← gets larger

By A.S.T. given series converges (we don't know yet : conditional or absolute)

Look @ Positive Part

$\sum \frac{1}{\sqrt[3]{2k+11}}$ For large k $\approx$ $\sum \frac{1}{\sqrt[3]{2k}} = \sum \frac{1}{(2k)^{1/3}} = \sum \frac{1}{2^{1/3} \cdot k^{1/3}} = \frac{1}{2^{1/3}} \sum \frac{1}{k^{1/3}}$

div. b/c $p = 1/3 < 1$

(1.5) $\sum_{k=1}^{\infty} \frac{7^k}{(2k)!}$

$(2(k+1))! = (2k+2)!$

Ratio Test:

$$\lim_{k \to \infty} \frac{\frac{7^{k+1}}{(2(k+1))!}}{\frac{7^k}{(2k)!}} = \frac{\cancel{7^k} \cdot 7}{(2k+2)\cdot(2k+1)\cancel{(2k)!}} \cdot \frac{\cancel{(2k)!}}{\cancel{7^k}} = \lim_{k \to \infty} \frac{7}{(2k+2)(2k+1)} = 0$$

Finite

$\Rightarrow$ series converges

Prove the following statement:

(1.6) If $\sum a_n$ converges, then $\lim_{n \to +\infty} a_n = 0$

10.6 Power Series

(infinite polynomials : $x+x^2+x^3+x^4+\dots$

$\frac{x}{1}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\dots$

$\sum_{n=0}^{\infty} a_n x^n$

centered @ $\emptyset$

$\sum_{n=0}^{\infty} a_n (x-c)^n$ centered @ c

Poly

x^2+x

$3x+5x^4+7$

Not

$\frac{1}{x}$

$x^{-2}+7$

$\sin(x)$

Most of the functions you (ever) seen can be expressed as power series

use Ratio Test to show the power series converges absolutely.

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = \sin(x)$$

Increment positive portion

take abs value

$$\lim_{n\to\infty} \frac{\frac{x^{2(n+1)+1}}{(2(n+1)+1)!}}{\frac{x^{2n+1}}{(2n+1)!}} = \frac{x^{\overbrace{2(n+1)+1}^{\overbrace{2n+2+1}^{2n+3}}}}{\underbrace{(2(n+1)+1)!}_{2n+3}} \cdot \frac{(2n+1)!}{x^{2n+1}}$$

$$\lim_{n\to\infty} \frac{x^{2n+3} \cdot \cancel{(2n+1)!}}{(2n+3)(2n+2)\cancel{(2n+1)!}\, x^{2n+1}} = \lim_{n\to\infty} \frac{x^2}{(2n+3)(2n+2)} = 0 < \infty$$

" $x^{2n} \cdot x^3$

$x^{2n} \cdot x$

converges