Math 163 - Calculus - Exam 2 - Guide **Name:** ____________________

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1.(1.1) Indicate absolute convergence, conditional convergence or divergence

$$\sum_{k=2}^{\infty} \frac{5k\sqrt{k}}{\underbrace{7k^3+5k+1}}$$

- large n $\approx \frac{5k\sqrt{k}}{7k^3} = \frac{5k^{3/2 = 1.5}}{7k^3} = \frac{5}{7k^{3/2}}$

- Direct Comparison: $\sum \frac{5}{7k^{3/2}}$ converges b/c it's a p-series $p = 3/2 > 1$

given

$$\frac{5k\sqrt{k}}{7k^3+5k+1} < \frac{5}{7k^{3/2}} \quad \forall \; k \geq 2$$

- Since the given series is bounded above by a convergent series, it too must converge (D.C.T.)

(1.2) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt[3]{2k+11}}$

see $(-1)^k \Rightarrow$ Alt. series

A.S.T. (1. Alt 2. decreasing, clearly $\frac{1}{\sqrt[3]{2k+11}}$ is decreasing. num = const, denom getting larger $\Rightarrow$ converges

$\Rightarrow$ given series converges by A.S.T.

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check for abs. conv.

$\sum \frac{1}{\sqrt[3]{2k+11}}$ For large n: $= \sum \frac{1}{\sqrt[3]{2k}}$ diverges p-series

L.C.T.

$$\lim_{k\to\infty} \frac{\frac{1}{\sqrt[3]{2k+11}}}{\frac{1}{\sqrt[3]{2k}}} = \lim_{k\to\infty} \frac{\sqrt[3]{2k}}{\sqrt[3]{2k+11}} = 1 \Rightarrow \text{behave same} \Rightarrow \text{diverges}$$

$\Rightarrow$ conditional converge

(1.5) $\sum_{k=1}^{\infty} \frac{7^k}{(2k)!}$

Ratio:

$$\lim_{k\to\infty} \frac{\frac{7^{k+1}}{(2(k+1))!}}{\frac{7^k}{(2k)!}} = \frac{7^{k+1}}{\underbrace{(2(k+1))!}_{(2k+2)!}} \cdot \frac{(2k)!}{7^k} = \frac{\cancel{7^k} \cdot 7 \cdot \cancel{(2k)!}}{\underbrace{(2k+2)(2k+1)}\cancel{(2k)!} \cdot \cancel{7^k}} = \frac{7}{(2k+1)^2} \to 0$$

$\Rightarrow$ converges

Prove the following statement:

(1.6) If $\sum a_n$ converges, then $\lim_{n\to+\infty} a_n = 0$

10.6 Power Series

infinite polynomials, eg, $x^2 + x$, $3x^4 + x + 1$

Not: $\frac{1}{x}$, $\sin(x)$, $x^{-1} + x$

$\sum_{n=0}^{\infty} a_n x^n$, centered @ $x=0$

$$\boxed{\sum_{n=0}^{\infty} a_n (x-c)^n \quad \text{centered @ } x=c}$$

Cool Fact: Most functions you've seen can be written (expressed) as a power series.

Closely related to geometric series

Ex $\sum_{n=0}^{\infty} c r^n$ $\quad$ $c =$ constant, $r =$ some ratio $\}$ converges when $|r| < 1$

Ex $\sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!}$

when does this converge absoutley? $\sum \frac{x^{2n+1}}{(2n+1)!} = \sum_{n=0} \left(\frac{1}{2n+1}\right)! \, x^{2n+1}$ $\quad$ Need some condition on the x.

if $|x| > 0 \Rightarrow$ converges (but more slowly the farther you are from center)

$x = 0 \Rightarrow$ converges to 0.

use ratio test to find what x's make this converge

$$\lim_{n\to\infty} \frac{\frac{x^{\overbrace{2(n+1)}^{2n+2}+1}}{(2(n+1)+1)!}}{\frac{x^{2n+1}}{(2n+1)!}} = \frac{x^{2n+3}}{\underset{\downarrow}{(2n+3)!}} \cdot \frac{(2n+1)!}{x^{2n+1}} = \frac{\overbrace{x^{2n+3}}^{x^{2n} \cdot x^3}}{(2n+3)(2n+2)\underbrace{x^{2n+1}}_{x^{2n} \cdot x}} = \frac{x^2}{(2n+3)(2n+2)} = 0$$

$(2n+3)(2n+2)(2n+1)!$

$\Rightarrow$ converge for all x