10.6 wk 8 Wed ______ **Power Series**: Infinite Polynomial

$$\sum_{n=0}^{\infty} a_n (x-c)^n$$

Two Conventions

$0^0 = 1$ $\qquad 2^0=1,\ 1^0=1,\ (\tfrac{1}{2})^0=1,\ (\tfrac{1}{4})^0=1, \dots$

$0! = 1$ $\qquad n! = n\cdot(n-1)!$

$3! = 3\cdot 2!$

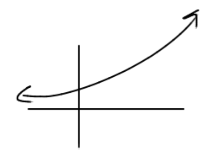
$1 = 1! = 1\cdot 0!$

Ex (e^x)

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \dots = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

since x is a variable, some choices of x may affect convergence

Question: For what values of x does this series converge?



Ratio Test $\Rightarrow$ ①

$$\left| \frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}} \right| = \frac{\overbrace{x^{n+1}}^{\cancel{x^n}\cdot x}}{\underbrace{(n+1)!}_{(n+1)\cancel{n!}}} \cdot \frac{\cancel{n!}}{\cancel{x^n}} = \left| \frac{x}{n+1} \right| \xrightarrow[n\to\infty]{\lim} 0 < 1$$

$\Rightarrow$ Converges for all x

I of Conv = $\mathbb{R}$

Ex

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{(n+2)3^n}$$ For what values does the series converge?

Ratio Test:

① $$\left| \frac{\dfrac{(x-2)^{n+1}}{(n+3)3^{n+1}}}{\dfrac{(x-2)^n}{(n+2)3^n}} \right| = \left| \frac{\overbrace{(x-2)^{n+1}}^{(x-2)\cancel{(x-2)^n}}}{(n+3)\underbrace{\cancel{3^{n+1}}}_{3\cdot\cancel{3^n}}} \cdot \frac{(n+2)\cancel{3^n}}{\cancel{(x-2)^n}} \right| = \left| \frac{(x-2)(n+2)}{(n+3)\cdot 3} \right| \xrightarrow[n\to\infty]{\lim} \boxed{\text{converges when } \left| \frac{x-2}{3} \right| < 1}$$

② solve inequality

$$-1 < \frac{x-2}{3} < 1$$

×3

$$-3 < x-2 < 3$$

+2

$$-1 < x < 5$$

interval notation

$$x \in (-1,5)$$

we know series converges whenever $x \in (-1,5)$

★ $x \in [-1,5)$

③ Test case by case basis to check on endpoints: $x=-1 \Rightarrow$ make $\left|\frac{x-2}{3}\right| = 1$; $x=5 \Rightarrow$ make $\left|\frac{x-2}{3}\right| = 1$ } inconclusive R.T.

$x=-1$ sub into O.G.

$$\sum_{n=0}^{\infty} \frac{(-1-2)^n}{(n+2)3^n} = \sum \frac{(-3)^n}{(n+2)\cancel{3^n}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+2}$$ where $(-3)^n = (-1)^n \cdot \cancel{3^n}$

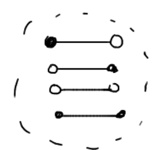
conv. by A.S.T. $\Rightarrow$ include $x=-1$ ★

$x=5$

$$\sum_{n=0}^{\infty} \frac{(5-2)^n}{(n+2)3^n} = \sum \frac{3^n}{(n+2)3^n} = \sum_{n=0}^{\infty} \frac{1}{n+2}$$ div. $\Rightarrow$ exclude $x=5$

(p = 1 series)

The "Interval of Convergence": may be



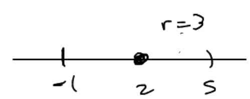
or $\mathbb{R}$ or •

The radius of convergence is half the length of the Interval of Conv.

e.g. $[-1,5)$ = Int. of Conv.

length $= 5-(-1) = 6$

Radius of Conv $= \frac{6}{2} = 3$



we'll see that when a series converges — we can

① "do algebra" on it

② "do calculus" on it (derivatives, integrals)

we'll see:

$$e^x = \sum \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$

d/dx

$$= 0 + 1 + \frac{2x^1}{2!} + \frac{3x^2}{3!} + \frac{4x^3}{4!} + \frac{5x^4}{5!}$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

same

e^x is the function that equals its own derivative