Interval of convergence

1. Find the interval of convergence for the following Taylor series:

$$\frac{1}{6-x} = \frac{1}{4} + \frac{1}{4^2}(x-2) + \frac{1}{4^3}(x-2)^2 + \cdots + \frac{1}{4^{n+1}}(x-2)^n + \cdots$$

"Apply" the ratio test . . . .

(i) Form the fraction $|a_{n+1}|/|a_n|$ and simplify as much as possible. (You need to include the power of x.)

$$\left|\frac{a_{n+1}}{a_n}\right| =$$

(ii) Evaluate the limit:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| =$$

(iii) Recalling that the limit in (ii) needs to be less than 1, find the *open* interval of convergence for the series, in the form $a < x < b$.

(iv) Now "check the endpoints" of the open interval you found for convergence.

2. Find the interval of convergence for the Taylor series:

$$e^{-x/3} = 1 - \frac{1}{3}\,x + \frac{1}{2!3^2}\,x^2 + \cdots + \frac{(-1)^n}{n!3^n}\,x^n + \cdots$$

"Apply" the ratio test . . . .

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(ii) Evaluate the limit:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| =$$

(iii) Recalling that the limit in (ii) needs to be less than 1, find the interval of convergence for the series.