Finding Taylor series the "easy" way . . . .

$$e^x = 1 + x + \frac{1}{2!}\,x^2 + \frac{1}{3!}\,x^3 + \cdots + \frac{1}{n!}\,x^n + \cdots +$$
$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots$$

1. Use the above to find the Taylor series at 0 for e^{2x^2}.

$$e^{2x^2} = 1 + 2x^2 + \frac{1}{2!}\,(2x^2)^2 + \frac{1}{3!}\,(2x^2)^3 + \cdots$$

$$e^{2x^2} = 1 + 2x^2 + \frac{1}{2!}\,4x^4 + \frac{1}{3!}\,8x^6 + \cdots$$
$$= \sum_{n=0}^{+\infty} \frac{2^n}{n!} x^{2n}$$

2. Use the above to find the Taylor series at 0 for $\frac{1}{1-8x^3}$.

$$\frac{1}{1-8x^3} = 1 + 8x^3 + (8x^3)^2 + (8x^3)^3 + \cdots + (8x^3)^n + \cdots$$
$$\frac{1}{1-8x^3} = 1 + 8x^3 + 64x^6 + 512x^9 + \cdots + 8^n x^{3n} + \cdots$$
$$= \sum_{n=0}^{+\infty} 8^n x^{3n}$$

3. Use the series on the previous page to find the Taylor series at 0 for

$$f(x) = \frac{x}{1-x^2} = x\left[\frac{1}{1-x^2}\right] = x\left[1 + x^2 + x^4 + x^6 + x^8 + \cdots\right]$$

$$= x + x^3 + x^5 + x^7 + x^9 + \cdots = \sum_{n=0}^{+\infty} x^{2n+1}$$

4. Use #3 to find the Taylor series at 0 for

$$g(x) = \frac{1+x^2}{(1-x^2)^2} = \frac{d}{dx}\left[\frac{x}{1-x^2}\right] = \frac{d}{dx}\left[x + x^3 + x^5 + x^7 + x^9 + \cdots\right]$$

$$= 1 + 3x^2 + 5x^4 + 7x^6 + 9x^8 + \cdots = \sum_{n=0}^{+\infty}(2n+1)x^{2n}$$

5. Find the intervals of convergence for

#1: $-\infty < x < +\infty$

#2: $-\frac{1}{2} < x < \frac{1}{2}$

#3: $-1 < x < 1$

#4: $-1 < x < 1$