Exam III, Chapter 8 & Sections 11.7 - 11.11

4. The Maclaurin series for the function $\sin x$ is shown below Carefully show that the interval of convergence for the series is $-\infty < x < +\infty$.

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

5. Find the fifth degree Taylor polynomial for the function $f(x) = \sin x + \cos x$.

6. Use the fifth degree Taylor polynomial from problem 5 to estimate $\sin 1 + \cos 1$.

7. Use a sixth degree Taylor polynomial to estimate

$$\int_0^1 \sin(x^2)\, dx$$

8. Find the Maclaruin series for the funciton $\tan^{-1} x$. (Derive it - either from Taylor formula (not recommended) or some other method. For example, the power series for the function $1/(1-x)$ might be helpful.)

Then show that the interval of convergence for the series is $[-1, 1]$.