Fri wk 9

Exam 3: week from Monday

Today: Taylor's Remainder thm. (10.8) (New HW Exercises: (Part 5))

Next Week Mon, Wed, thur. } Arc length, Parametric Curves

Fri → Review

Study Guide: Mon/Tue

Taylor's Remainder Thm.

For $f(x) = 1 + 3x^2$, $f(0) = 1$, $f'(x) = 6x$, $f'(0) = 0$

Then $T_1(x) = f^{(0)}(0) \cdot \frac{(x-0)^0}{0!} + f^{(1)}(0) \cdot \frac{(x-0)^1}{1!} = 1 \cdot \frac{1}{1} + 0 \cdot \frac{(x-0)^1}{1!} = 1$

1^{st} order Taylor Approx to $f(x)$ @ $a=0$, centered @ 0

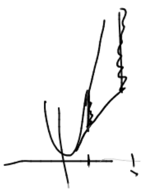
$n=1$

Taylor Polynomial

$$\sum_{n=0}^{\infty} \underbrace{f^{(n)}(a)}_{n^{th} \text{ derivative @ } x=a} \frac{(x-a)^n}{n!}$$

If $|f^{(n+1)}(x)| \leq M$ for all x s.t. $|x-a| < d$

" for all x d-close to a"



then

$$\boxed{|R_n(x)| \leq \frac{M \cdot |x-a|^{n+1}}{(n+1)!}}$$

$|R_n(x)| = \text{Error}(x)$

for all x s.t. $|x-a| < d$

$\approx |R_n(x)| \leq f^{(n+1)}(a^*) \frac{|x-a|^{n+1}}{(n+1)!}$

a^* = some x-value where $f^{(n+1)}$ peaks

Ex Use Taylor series to estimate $e^{1.5}$ up to 10^{-3} accuracy.
on $[-2,2]$
(since $1.5 \in [-2,2]$

$e^{1.5} \approx$ ____

① Start w/ $a=0$, (MacLaurin).

How big is N s.t. $e^{1.5}$ is close to true val.
small error.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad e^x \approx \sum_{n=0}^{N} \frac{x^n}{n!}$$

② Tay! Rem Thm: $|R_n(x)| \leq \dfrac{M x^{n+1}}{(n+1)!}$

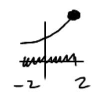
here $x = 1.5$
M needs to be: $M \geq f^{(n+1)}(x)$ on $[-2,2]$

our error @ $x=1.5$: $|R_n(1.5)| \leq \dfrac{9 \cdot (1.5)^{n+1}}{(n+1)!} < 10^{-3}$

solve

use: $e < 3$

so



$e^2 < 3^2 = 9$

so $M=9$ works. $9 > e^x$ for $x \in [-2,2]$

$(5+1)! = 6! = 6 \cdot 120 = 720$

$$9000\,(1.5)^{n+1} < (n+1)!$$

$n = 5 \Rightarrow$

$n = 10 \Rightarrow 11! =$

|| need 9 $\Rightarrow n = 8$

$$\sum_{n=0}^{8} \frac{(1.5)^n}{n!} \approx e^{1.5} \quad \text{w/ in } 10^{-3}$$