Mon Wk 9

See: Achieve

Today: Euler's Formula, $\sum \frac{f^{(n)}(0)}{n!} x^n$

Binomial Maclaurin

Start: Maclaurin Series:

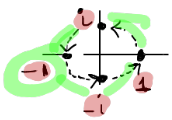
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad (*)$$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n}}{(2n)!}$$

In physics / engineering , complex numbers are used together with Taylor / Maclaurin

Need to know:



$i^2 = (\sqrt{-1})^2 = -1$

$i^3 = -1 \cdot i$

$i^4 = (-1)(-1) = 1$

multiplication by $i\ (\sqrt{-1})$ $\updownarrow$ rotation

Also For convergent series, reording of terms is valid.

Turns out, subtituting i, or $i\theta$ into e^x gives convergent series.

i.e., HW Part 4:

$$\sum_{n=0}^{\infty} \frac{(\ln 6)^n}{n!} = e^{(\ln 6)} = 6 \quad \text{b/c } (*)$$

$$e^{i\theta} = \sum_{n=0}^{\infty} \frac{(i\theta)^n}{n!}$$

$$= \underset{n=0}{1} + \frac{(i\theta)^1}{1} + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \frac{(i\theta)^6}{6!} + \frac{(i\theta)^7}{7!} + \dots$$

$$= 1 + \frac{\overset{-\theta^2}{(i\theta)^2}}{2!} + \frac{\overset{=\theta^4}{(i\theta)^4}}{4!} + \frac{\overset{=-\theta^6}{(i\theta)^6}}{6!} + \dots + i\theta + \frac{\overset{=-i\theta^3}{(i\theta)^3}}{3!} + \frac{\overset{=i^4 \cdot i}{(i\theta)^5}}{5!} + \frac{(i\theta)^7}{7!} + \dots$$

even powered, odd powered

$$= \underbrace{1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots}_{\cos\theta} + \underbrace{i\theta - \frac{i\theta^3}{3!} + \frac{i\theta^5}{5!} - \frac{i\theta^7}{7!} + \dots}_{i\left(\underbrace{\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots}_{\sin\theta}\right)}$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

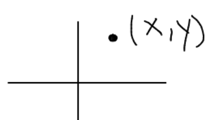
Exercise: substitute pi above, to produce an equation that relates 5 of the most important numbers in math

$$e^{i\pi} = \cos\pi + i\sin\pi$$

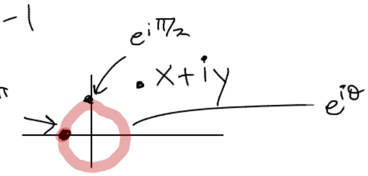
$$e^{i\pi} = -1 + i(0) = -1$$

$$\boxed{e^{i\pi} + 1 = 0}$$

Further Connection



Cartesian Plane $\longleftrightarrow$



Complex Plane

Binomial Expansion (MacLaurin) (HW Part I, #5)

$$(1+x)^{\frac{1}{2}} = 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^3 + \tfrac{1}{16}x^4$$

$n = \frac{1}{2}$ $\quad \binom{n}{0} \quad \binom{n}{1} \quad \binom{n}{2} \quad \binom{n}{3} = \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} = \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!} = \frac{\frac{3}{8}\cdot\frac{1}{6}}{} \quad \frac{3}{48} = \frac{1}{16}$

$\binom{n}{1} = \frac{\frac{1}{2}}{1!}$

$\binom{n}{2} = \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} = \frac{\frac{1}{2}\cdot(-\frac{1}{2})}{2!}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

n $\quad k$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n\cdot(n-1)(n-2)\cdots\cancel{(n-k)!}}{k!\cancel{(n-k)!}}$$

coefficients

$$= \frac{n(n-1)(n-2)\cdots(n-(k-1))}{k!}$$

MacLaurin Series

$$\frac{f^{(n)}(0)}{} = \sum \frac{f^{(n)}(0)}{n!}x^n$$

$f(x) = (1+x)^{\frac{1}{2}}$ | $f(0) = 1$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$ | $f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2}$ | $f'(0) = -\frac{1}{4} \rightarrow \frac{-\frac{1}{4}}{2!} = -\frac{1}{8}$