Power Series

$f(x) = \sum_{n=0}^{\infty} c_n x^n$ centered @ $x = 0$ center point where it's "accurate"!

$= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots$

comes from

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots = \sum_{n=0}^{\infty} x^n$$

works for x near 0.

$\boxed{|x| < 1 \text{ geometric series}}$

integrate

$$\int \frac{1}{1-x}\,dx$$

$u = 1 - x$
$du = -dx$
$-du = dx$

$$= \int \frac{-du}{u} = -\ln|u| = -\ln(u)$$

b/c $|x| < 1$ $1 - x > 0$
so drop abs value :

$$= -\ln(u) = -\ln(1-x)$$
$$= \ln(1-x)^{-1} = \ln\left(\frac{1}{1-x}\right) =$$

$$\ln\left(\frac{1}{1-x}\right) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} + \ldots +$$

Interval of Converge

$$= \sum_{n=1}^{\infty} \frac{x^n}{n}$$

Ratio Test

$$\left| \frac{\frac{x^{n+1}}{n+1}}{\frac{x^n}{n}} \right| = \left| \frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n} \right| = \left| \frac{x^n \cdot x \cdot n}{(n+1)\, x^n} \right|$$

if $\lim_{n \to \infty} \left| \frac{x \cdot n}{n+1} \right| = |x| < 1$

it converges

our interval is $(-1, 1)$

$$\ln\left(\frac{1}{1-x}\right) = \sum_{n=1}^{\infty} \frac{x^n}{n} \quad \text{when } x \in (-1, 1)$$

what about $x = -1$?

plug in

$[-1, 1)$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \longrightarrow \text{Converges} \quad (x = -1 \text{ works})$$

$x = 1$?

$$\sum_{n=1}^{\infty} \frac{1}{n} \longrightarrow \text{Diverges} \quad (x = 0)$$