Thur. Wk 9
Homework Ex.

HW Part 4 #6

$n=0 \leftrightarrow 2 = k\cdot 0 + 2$
$n=1 \leftrightarrow 6 = k\cdot 1 + 2$
$n=3 \leftrightarrow 10$

$6 = k\cdot 1 + 2$
$k=4$

$$\int_0^1 2\tan^{-1}(x^2)\,dx$$

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots = \overbrace{\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}}^{=\tan^{-1}x}$$

sub $x \sim x^2$

$$= \sum_{n=0}^{\infty}(-1)^n \frac{(x^2)^{2n+1}}{2n+1}$$

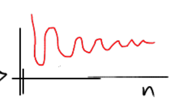
$$\tan^{-1}(x^2) = x^2 - \frac{(x^2)^3}{3} + \frac{(x^2)^5}{5} - \frac{(x^2)^7}{7} = x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \frac{x^{14}}{7} + \dots = \sum_{n=0}^{\infty}(-1)^n \frac{x^{4n+2}}{2n+1}$$

$$\int_0^1 2\tan^{-1}(x^2)\,dx = \int_0^1 2\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n+2}}{2n+1}\,dx = 2\sum_{n=0}^{\infty}(-1)^n \int_0^1 \frac{x^{4n+2}}{2n+1}\,dx = 2\sum_{n=0}^{\infty}(-1)^n \frac{1}{2n+1}\int_0^1 x^{4n+2}\,dx$$

$$= 2\sum_{n=0}^{\infty}(-1)^n \frac{1}{2n+1}\left.\frac{x^{4n+3}}{4n+3}\right|_0^1 = 2\sum(-1)^n \frac{1}{2n+1}\left[\frac{1}{4n+3} - \frac{0}{4n+3}\right]$$

$$= \sum_{n=0}^{\infty}(-1)^n \frac{2}{(2n+1)(4n+3)}$$

want to find appx. ans $\int$. Key: ALT. SERIES $\Rightarrow$

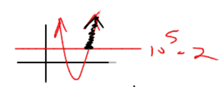


Error in the n-th appx is bounded by the (n+1) term

We want the error $< 10^{-5}$

$$\text{error} \le \frac{2}{(2(n+1)+1)(4(n+1)+3)} < 10^{-5} \quad \text{solve for } n$$

$$\frac{1}{(2n+3)(4n+7)} < 10^{-5} \Rightarrow 2\cdot 10^5 < (2n+3)(4n+7) = 8n^2 + 26n^2 + 21$$



$n=157 \Rightarrow$ makes ineq true

$$\sum_{n=0}^{157} \frac{(-1)^n 2}{(2n+1)(4n+3)} = .598003$$

Hw Part 4 # 4 & 5

#5/

$$\lim_{x\to 0} \frac{\tan^{-1}(7x) - 7x\cos(7x) - \frac{343}{6}x^3}{x^5} = \lim \frac{\overbrace{\sum \sim - \sum \sim}^{\sum \sim} \quad 343x^3}{}$$

~~1. Expand JUST using summation notation first.~~
2. Pull out the first few terms so that when we divide by x^5, we avoid dividing by zero.

$$\tan^{-1}(7x) = 7x - \frac{(7x)^3}{3} + \frac{(7x)^5}{5} - = \sum_{n=0}^{\infty} (-1)^n \frac{(7x)^{2n+1}}{2n+1}$$

$$7x\underbrace{\cos(7x)}_{*} = 7x\left(1 - \frac{(7x)^2}{2!} + \frac{(7x)^4}{4!} - \dots\right) = 7x \cdot \underbrace{\sum_{n=0}^{\infty} (-1)^n \frac{(7x)^{2n}}{(2n)!}}_{*} = \sum_{n=0}^{\infty} (-1)^n \frac{\cdot 7x^{2n+1}}{(2n)!}$$

$$\tan^{-1}(7x) - 7x\cos(7x) = \sum_{n=0}^{\infty} (-1)^n \frac{(7x)^{2n+1}}{2n+1} - \sum_{n=0}^{\infty} (-1)^n \frac{\cdot 7x^{2n+1}}{(2n)!} = \sum_{n=0}^{\infty} (-1)^n \frac{7x^{2n+1}}{2n+1} - \frac{7x^{2n+1}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n (7x)^{2n+1} \left(\frac{1}{2n+1} - \frac{1}{(2n)!}\right)$$

$=0$

$$7x\left(\overbrace{\frac{1}{2\cdot 0+1} - \frac{1}{0!}}^{=0}\right)$$

$$(-1)(7x)^3 \cdot \left(\underline{\frac{1}{3} - \frac{1}{2}}\right)$$

$$-7^3x^3\left(-\frac{1}{6}\right) = 7^3x^3$$

$$+ \underbrace{\sum_{n=3}^{\infty} (-1)^n (7x)^{2n+1} \left(\frac{1}{2n+1} - \frac{1}{(2n)!}\right)}_{\text{here you can sub } x=0.}$$

$n=0$ $n=1$ $n=2$