MA163 wed

Exam 3 : March 31

HW 10, Part I

Question 4 of 8

Compute T_3 for $f(x) = 3\sqrt{x}$ centered at $a = 1$.

(Use symbolic notation and fractions where needed.)

$$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} \cdot (x-a)^n$$

$f(x) = 3\sqrt{x}$ $\quad f(1) = 3$

$f'(x) = \frac{3}{2} x^{-1/2}$ $\quad f'(1) = \frac{3}{2}$

$f''(x) = -\frac{3}{4} x^{-3/2}$ $\quad f''(1) = -\frac{3}{4}$

$f'''(x) = \frac{9}{8} x^{-5/2}$ $\quad f'''(1) = \frac{9}{8}$

$$T_3(x) = \frac{3}{0!}(x-1)^0 + \frac{\frac{3}{2}}{1!}(x-1)^1 + \frac{-\frac{3}{4}}{2!}(x-1)^2 + \frac{\frac{9}{8}}{\underset{6}{3!}}(x-1)^3$$

$$= 3 + \frac{3}{2}(x-1) - \frac{3}{8}(x-1)^2 + \frac{9}{8 \cdot 6}(x-1)^3$$

$$= 3 + \frac{3}{2}(x-1) - \frac{3}{8}(x-1)^2 + \frac{3}{16}(x-1)^3$$

Use a plot of the error $\overbrace{|f(x) - T_3(x)|}^{\text{function}}$ to find the largest value $c > 1$ such that the error on the interval $[1, c]$ is at most 0.75.

(Use decimal notation. Give your answer to three decimal places.)

$c =$

$$E(x) = \left| 3\sqrt{x} - \left[3 + \frac{3}{2}(x-1) - \frac{3}{8}(x-1)^2 + \frac{3}{16}(x-1)^3\right] \right|$$

HW Part 4 #6

Express the definite integral as an infinite series in the form $\sum_{n=0}^{\infty} a_n$.

$$\int_0^1 2\tan^{-1}(x^2)\,dx$$

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$a_n =$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$$

sub $(x \sim (-x^2))$

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 + \dots$$

integrate

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots$$

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

$$\xrightarrow{@\, x \sim (x^2)} \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{2n+1}}{2n+1} = \sum (-1)^n \frac{x^{4n+2}}{2n+1}$$

$$\tan^{-1}(x^2) = \underbrace{x^2}_{n=0} - \underbrace{\frac{x^6}{3}}_{n=1} + \underbrace{\frac{x^{10}}{5}}_{n=2} - \frac{x^{14}}{7} + \frac{x^{18}}{9} - \dots + = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{2n+1}$$

$k=4$
- $kn + 2 = 2 \longleftrightarrow 0 = n$
- $kn + 2 = 6 \longleftrightarrow 1 = n$
- $k2 + 2 = 10 \longleftrightarrow 2 = n$

$$\int_0^1 2\left(x^2 - \frac{x^6}{3}\right) = 2\left[\frac{x^3}{3} - \frac{x^7}{21}\right]\Bigg|_0^1$$

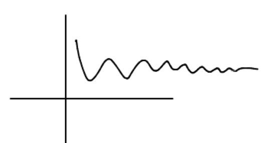
$$\int_0^1 2\tan^{-1}x\,dx = \int_0^1 2\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n+2}}{2n+1}\,dx = 2\int_0^1 \sum (-1)^n \frac{x^{4n+2}}{2n+1}\,dx = 2\sum_{n=0}^{\infty}\int_0^1 (-1)^n \frac{x^{4n+2}}{2n+1}\,dx$$

$$= 2\sum_{n=0}^{\infty}(-1)^n \int_0^1 \frac{x^{4n+2}}{2n+1}\,dx = 2\sum_{n=0}^{\infty}(-1)^n \frac{1}{2n+1}\underbrace{\int_0^1 x^{4n+2}\,dx}_{\frac{x^{4n+3}}{4n+3}} = 2\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n+3}}{(2n+1)(4n+3)}\Bigg|_0^1$$

$$= 2\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n+1)(4n+3)} - \frac{0}{(2n+1)(4n+3)} = \sum_{n=0}(-1)^n \frac{2}{(2n+1)(4n+3)}$$

this is an alt. series, which are easy to approximate!

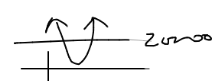
Error incurred by approximating using only n terms of an alternating series is less than the (n+1) term



sub $(n+1)$

$$\frac{2}{(2(n+1)+1)(4(n+1)+3)} = \frac{2}{(2n+3)(4n+7)} < 10^{-5}$$

set / solve this $\longrightarrow < 10^{-5}$



$$200000 = 2 \cdot 10^5 = \frac{2}{10^{-5}} < (2n+3)(4n+7) = 8n^2 + 26n + 21$$

say @ $n = 100$ this happens,

$$\int_0^1 2\tan^{-1}x = \sum_{n=0}^{100} (-1)^n \frac{2}{(2n+1)(4n+3)}$$