MA 211 Concept Check Up

linear comb.

$\vec{v}_1 = -\frac{c_2}{c_1}\vec{v}_2 - \frac{c_3}{c_1}\vec{v}_3 - \dots - \frac{c_n}{c_1}\vec{v}_n$

Lin. Ind = no lin. combo of some equals the other

$c_1v_1 + c_2v_2 + \dots + c_nv_n = 0$ has only trivial soln.

1. Define the following:

(a) basis for a subspace of $\mathbb{R}^n$

(b) span of v_1, v_2, v_3 in $\mathbb{R}^n$.

(c) linear independence of a set of vectors $\{v_1, v_2, \dots, v_n\}$

(d) column space of a matrix "col(A)"

(e) row space of a matrix "row(A)"

(f) null space of a matrix "null(A)" all solns to $A\vec{x} = \vec{0}$.

(g) null space of matrix transpose null(A^T). all sols to $A^T\vec{x} = \vec{0}$

2. Let $A = \begin{pmatrix} 1 & -1 & 4 \\ 2 & 4 & 6 \\ 3 & 3 & 10 \end{pmatrix}$ Compute bases for the four fundamental subspaces of A.

row(A)	col(A)	null(A)	null(A^T)

3. Verify that null(A) ⊥ row(A).

4. Verify that null($A^\perp$) ⊥ col(A).

5. Verify that dim(row(A)) = dim(col(A)) (7.5 Rank Theorem rank(A) = rank(A^T))

6. Verify that rank(A) + nullity(A) = number of columns of A

dim(row(A))

7. Let $A = \begin{pmatrix} 2 & -5 & 7 \\ 0 & 8 & 0 \\ 0 & 0 & 12 \end{pmatrix}$ Compute bases for the four fundamental subspaces of A.

3 basis veks

row(A)	col(A)	null(A)	null(A^T)
		$\{\vec{0}\}$	$\{\vec{0}\}$

8. Verify that null(A) ⊥ row(A).

9. Verify that null($A^\perp$) ⊥ col(A).

10. Verify that dim(row(A)) = dim(col(A)) (7.5 Rank Theorem

11. Verify that rank(A) + nullity(A) = number of columns of A

$A = \begin{pmatrix} 1 & -1 & 4 \\ 2 & 4 & 6 \\ 3 & 3 & 10 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 4 \\ 0 & 6 & -2 \\ 0 & 0 & 0 \end{pmatrix}$ $-2R_1 + R_2$

Row(A): Basis = {(1, −1, 4), (0, 6, −2)}.

Row(A) = span({(1, −1, 4), (0, 6, −2)})

#3 $(1,-1,4)\cdot(-11,1,3) = -12+12 = 0$

$(0,6,-2)\cdot(-11,1,3) = 0$.

Basis: $\begin{pmatrix} -11 \\ 1 \\ 3 \end{pmatrix}$

$\boxed{\text{null}(A) = t\begin{pmatrix} -11 \\ 1 \\ 3 \end{pmatrix}}$

$\left(\begin{array}{ccc|c} 1 & -1 & 4 & 0 \\ 0 & 6 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$z = t$

$6y - 2t = 0$

$y = \frac{1}{3}t$

$x - (\frac{1}{3}t) + 4t = 0$

$x = -\frac{11t}{3}$

$\bar{x} = t\begin{pmatrix} -11/3 \\ 1/3 \\ 1 \end{pmatrix} = t\begin{pmatrix} -11 \\ 1 \\ 3 \end{pmatrix}$

$\forall t \in \mathbb{R}$

col(A) = row(A^T)

$\begin{pmatrix} 1 & 2 & 3 \\ -1 & 4 & 3 \\ 4 & 6 & 10 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 6 & 6 \\ 0 & -2 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

(−4, −8, −12)

basis = (1, 2, 3), (0, 1, 1)

null(A^T) = $t\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

dot products are zero

Rank Thm Ex.

$\begin{pmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 0 \\ 4 & 0 \\ 5 & 0 \end{pmatrix}$

$\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} \begin{pmatrix} 3 \\ 6 \end{pmatrix} \begin{pmatrix} 4 \\ 8 \end{pmatrix}\right)$

1 lin ind row

1 lin ind col

12 Let $A = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 5 & 10 \end{pmatrix}$ // Compute bases for the four fundamental subspaces of A. $\mathbb{R}^2$

row(A)	col(A)	null(A)	null(A^T)
$\text{span}(\{(1,-1,2),(1,5,10)\})$	$\text{span}\left(\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}-1\\5\end{pmatrix}\right) = \text{span}\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right\} = \mathbb{R}^2$	$= \text{span}\begin{pmatrix}-10\\-4\\3\end{pmatrix}$	$= \left\{\begin{pmatrix}0\\0\end{pmatrix}\right\}$

13 Verify that null(A) ⊥ row(A).

14 Verify that null(A^T) ⊥ col(A).

15 Verify that dim(row(A)) = dim(col(A))

16 Verify that rank(A) + nullity(A) = number of columns of A

17 Let $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ Compute bases for the four fundamental subspaces of A.

row(A)	col(A)	null(A)	null(A^T)

18 Verify that null(A) ⊥ row(A).

19 Verify that null(A^T) ⊥ col(A).

20 Verify that dim(row(A)) = dim(col(A))

21 Verify that rank(A) + nullity(A) = number of columns of A

$A\vec{x} = \vec{0}$

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 1 & 5 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 6 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$z = t$

$6y = -8t$

$y = -\frac{4}{3}t$

$x = y - 2t$

$= -\frac{4}{3}t - \frac{6t}{3}$

$= \frac{-10t}{3}$

$$\vec{x} = t\begin{pmatrix} -\frac{10}{3} \\ -\frac{4}{3} \\ 1 \end{pmatrix} = t\underbrace{\begin{pmatrix} -10 \\ -4 \\ 3 \end{pmatrix}}_{\text{basis}}$$

$A^T = \begin{pmatrix} 1 & 1 \\ -1 & 5 \\ 2 & 10 \end{pmatrix} \xrightarrow{\text{rref}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$

A^T sends $\mathbb{R}^2 \to \mathbb{R}^3$

$\Rightarrow$ Null(A^T) always lives in domain

$x = 0$

$y = 0$

$\vec{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$