Scalar Multiplication: $5v$ symbolic

$5(1,2,3) = (5,10,15)$

↑ scalar

Difference Vector

$u - v$

$\mathbb{R}^n$ = set of all n-tuples. $\underbrace{(x_1, x_2, x_3, \dots, x_n)}_{n \text{ real \#'s}}$

Linear Combination:

$\bar{u}, \bar{v}, \bar{w}$ 3 given vectors

any vector of the form:

$\bar{a} = c_1\bar{u} + c_2\bar{v} + c_3\bar{w}$

scalars

Ex.

$3(0,1) + \pi(1,0) = (\pi, 3)$

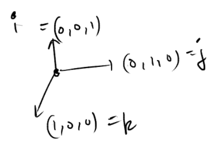
$(4.3, -7.9)$

$4.3(1,0) - 7.9(0,1) = (4.3, -7.9)$

Norm: (Magnitude) or Length

$\bar{v} = (v_1, v_2, \dots, v_n)$

$\|\bar{v}\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$



— STANDARD UNIT VECTORS in $\mathbb{R}^3$ — length 1.

$\|\vec{i}\| = \sqrt{0^2 + 0^2 + 1^2} = 1$

Dot Product: Produces a real # from 2 vectors

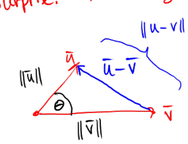
$\bar{u} = (u_1, u_2, u_3)$, $\bar{v} = (v_1, v_2, v_3)$

#'s

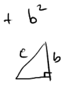
$\bar{u} \cdot \bar{v} = u_1v_1 + u_2v_2 + u_3v_3$ } real #

dot product

Surprise! The angle can be found using dot product.



$a^2 + b^2 = c^2$



90° ⇒ $\cos 90° = 0$

more generally

$\|\bar{u}\|^2 + \|\bar{v}\|^2 - 2\|\bar{u}\|\|\bar{v}\|\cos\theta = \|\bar{u}-\bar{v}\|^2$

$\boxed{a^2 + b^2 - 2ab\cos\theta = c^2}$

Fact: NORM-SQUARED of u IS DOT PRODUCT u·u

cancel

$(\bar{u}-\bar{v})(\bar{u}-\bar{v})$

$\bar{u}\cdot\bar{u} - \bar{u}\cdot\bar{v} - \bar{v}\cdot\bar{u} + \bar{v}\cdot\bar{v}$

$\|\bar{u}\|^2 \quad - 2\bar{u}\bar{v} \quad + \|\bar{v}\|^2$

$\cos\theta = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\cdot\|\bar{v}\|}$

$\theta = \cos^{-1}\left(\dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}\right)$

The angle b/w vectors:

Q: Is $\|\bar{u}\| \geq 0$? Yes.

$\|\bar{u}\| = \sqrt{\sum u_i \cdot u_i}$

Fact: $\|k\bar{v}\| = |k| \cdot \|\bar{v}\|$

Q: How can you tell if the angle b/w two vectors is acute or obtuse or 90°.

$u = (1, 2, 3)$

$v = (-1, 0, 5)$

$\theta = \cos^{-1}\left(\frac{u \cdot v}{\|u\|\|v\|}\right)$

$u \cdot v = 0 \Rightarrow \cos^{-1}(0) = 90^\circ$

$u \cdot v < 0 \Rightarrow$ obtuse

$u \cdot v > 0 \Rightarrow$ actute

Algebraic Properties of Dot.

- $\emptyset \cdot \bar{v} = \bar{\emptyset}$ $\qquad \emptyset(1,2,3,4,5,-1) = (0,0,0,0,0,0)$
- $(\bar{u}+\bar{v})\cdot\bar{w} = \bar{u}\cdot\bar{w} + \bar{v}\cdot\bar{w}$
-

Orthogonal vectors & Orthonormal Vectors

fancy for perpendicular

An orthogonal set:

$\{\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_n\}$

are all $\perp$ (orthogonal)

$\Rightarrow \bar{u}_1 \cdot \bar{u}_2 = \emptyset$

$\bar{u}_i \cdot \bar{u}_j = 0 \ \forall\, i,j$

for all

NORMAL means unit length

so to normalize means to scale s.t. length = 1.

Exercise

$v = (1,2,3)$

produce a vector pointing in same direction as $\bar{v}$, but having unit length.

$$\boxed{\bar{w} = \frac{1}{\|\bar{v}\|}\cdot\bar{v}}$$

kills the length.

vector