

MA211 HW 2.2 # 6, 11, 12, 24, 36, 38, 50

#6  $\begin{bmatrix} 1 & -7 & 5 & 5 \end{bmatrix}$  in RREF

$$\#11 \begin{bmatrix} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} x_5 = t \text{ (free)} \\ x_4 + x_5 = 8 \Rightarrow x_4 = 8 - t \\ x_3 + x_4 = 7 \Rightarrow x_3 + 4x_5 = 7 \\ x_3 = 7 - 4t \\ x_2 = r \text{ (free)} \\ x_1 - 6x_2 + 3x_5 = -2 \\ \Rightarrow x_1 = 6r - 3t - 2 \end{array}$$

$$\begin{aligned} \text{vector form: } & (6r - 3t - 2, r, 7 - 4t, 8 - t, t) \\ & = (-2, 0, 7, 8, 0) + r(6, 1, 0, 0, 0) + t(-3, 0, -4, -1, 1) \end{aligned}$$

#12  $\begin{pmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \text{no solutions, since the last row implies } 0 \cdot x_2 = 1. \quad$

$$\#24 \left. \begin{array}{l} 2x_1 + 2x_2 + 2x_3 = 0 \\ -2x_1 + 5x_2 + 2x_3 = 1 \\ 8x_1 + x_2 + 4x_3 = -1 \end{array} \right\} \begin{pmatrix} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{pmatrix} \begin{array}{l} \frac{1}{2}R_1 \\ R_1 + R_2 \\ -4R_1 + R_3 \end{array} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} R_2 + R_3$$

$$\rightarrow \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow x_3 = t \text{ (free)}, x_2 = 1/7 - 4/7t \\ x_1 + x_2 + x_3 = 0 \\ x_1 + 1/7 - 4/7t + t = 0$$

$$x_1 = -1/7 - 3/7t$$

$$\boxed{\bar{x} = \left( -1/7, 1/7, 0 \right) + t \left( -3/7, -4/7, 1 \right)}$$

$$\#36 \begin{array}{l} 3x_1 + x_2 + x_3 + x_4 = 0 \\ 5x_1 - x_2 + x_3 - x_4 = 0 \end{array} \rightarrow \begin{pmatrix} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -5 & -1 & -5 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{pmatrix} \xrightarrow{10R_2 - 3R_1} \begin{pmatrix} 1 & -5 & -1 & -5 & 0 \\ 0 & -24 & -6 & -24 & 0 \end{pmatrix} \xrightarrow{5R_1 - R_2}$$

$$\rightarrow \begin{pmatrix} 1 & -5 & -1 & -5 & 0 \\ 0 & 1 & 1/4 & 1 & 0 \end{pmatrix} \xrightarrow{-1/24R_3}$$

$$\Rightarrow x_4 = t \text{ (free)} \quad \frac{1}{4}x_2 + \frac{1}{4}x_3 + x_4 = 0 \Rightarrow x_2 = -t - \frac{1}{4}r \\ x_3 = r \text{ (free)} \quad x_1 - 5x_2 - x_3 - 5x_4 = 0$$

$$\Rightarrow x_1 = 5t + r + 5(-t - \frac{1}{4}r)$$

$$= r - 5/4r = -1/4r$$

$$\boxed{\bar{x} = \left( -1/4r, -t - 1/4r, r, t \right) = r \left( -1/4, 0, 1, 0 \right) + t \left( 0, -1, 0, 0, 1 \right)}$$

#38  $\begin{cases} 2x - y - 3z = 0 \\ -x + 2y - 3z = 0 \\ x + y + 4z = 0 \end{cases} \left\{ \begin{pmatrix} 2 & -1 & -3 & 0 \\ -1 & 2 & -3 & 0 \\ 1 & 1 & 4 & 0 \end{pmatrix} \right.$

 $\rightarrow \begin{pmatrix} 1 & 1 & 4 & 0 \\ 0 & 3 & 1 & 0 \\ 2 & -1 & -3 & 0 \end{pmatrix} \begin{array}{l} R_1 \leftrightarrow R_3 \\ R_3 + R_2 \end{array} \rightarrow \begin{pmatrix} 1 & 1 & 4 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & -3 & -7 & 0 \end{pmatrix} \begin{array}{l} -R_1 + R_3 \\ R_2 + R_3 \end{array}$ 
 $\rightarrow \begin{pmatrix} 1 & 1 & 4 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 1 & 7/3 & 0 \end{pmatrix} \begin{array}{l} R_2 + R_3 \\ R_2 \leftrightarrow R_3 \\ \frac{1}{6}R_3 \end{array} \Rightarrow \begin{pmatrix} 1 & 1 & 4 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ 
 $\Rightarrow x=0, y=0, z=0 \Rightarrow \bar{x} = (0, 0, 0)$

#50  $\begin{cases} x^2 + y^2 + z^2 = 6 \\ x^2 - y^2 + 2z^2 = 2 \\ 2x^2 + y^2 - z^2 = 3 \end{cases} \left\{ \begin{array}{l} x + y + z = 6 \\ x - y + 2z = 2 \\ 2x + y - z = 3 \end{array} \right.$

 $\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 2 \\ 2 & 1 & -1 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 2 & -1 & 4 \\ 0 & -1 & -3 & -9 \end{pmatrix}$ 
 $\rightarrow \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 3 & 9 \\ 0 & 2 & -1 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 3 & 9 \\ 0 & 0 & -7 & -14 \end{pmatrix}$ 
 $\rightarrow \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 3 & 9 \\ 0 & 0 & 1 & 2 \end{pmatrix} \Rightarrow \begin{array}{l} z = 2 \\ y + 3z = 9 \Rightarrow y = 9 - 3z = 3 \\ x + y + z = 6 \end{array}$ 
 $x = 6 - 3 - 2$ 
 $x = 1$ 
 $\Rightarrow (x, y, z) = (1, 3, 2)$ 

so  $(x^2, y^2, z^2) = (1, 3, 2) \Rightarrow \begin{array}{l} x = \pm 1 \\ y = \pm \sqrt{3} \\ z = \pm \sqrt{2} \end{array}$

2.3 D1  
Family of 2nd degree polys that pass thru  
 $(0,1) \nparallel (1,2)$ .

Start with general:  $ax^2 + bx + c = y$   
plug in  $(0,1) \Rightarrow 0 + 0 + c = 1$   
 $\frac{1}{2} (1,2) \Rightarrow a(1)^2 + b(1) + 1 = 2$   
or  $a + b + 1 = 2$   
 $a + b = 1$   
 $a = 1 - b$

so  $y = (1-b)x^2 + bx + 1$  is a poly that passes thru  $(0,1) \nparallel (1,2)$

for  $b=0 \Rightarrow y = x^2 + 1$   
example  $b=-1 \Rightarrow y = 2x^2 - x + 1 \} \text{ all work.}$