3.2

FIND A^{-1}: (12a) $A=\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -3 & 2 & -2 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] -2R_1+R_2$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 2 & -1 & -1 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{pmatrix} -2R_3-R_2$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 0 & 6 & -3 & 2 & 2 \\ 0 & 1 & -3 & 2 & -1 & -1 \\ 0 & 0 & 7 & -4 & 2 & 2.5 \end{pmatrix} \begin{array}{l} -2R_2+R_1 \\ \\ -2R_2+R_3 \end{array}$$

$$\downarrow$$

$$\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \underbrace{\begin{array}{ccc} .4286 & .2857 & -.5714 \\ .2857 & -.1429 & .2857 \\ -4/7 & 2/7 & 2.5/7 \end{array}}_{A^{-1}}\right) \begin{array}{l} -\frac{6}{7}R_3+R_1 \\ \frac{3}{7}R_3+R_2 \\ \frac{1}{7}R_3 \end{array}$$

(14) $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 4 & 8 & 0 \end{pmatrix} = A$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 4 & 0 & 0 & 1 & 0 \\ 4 & 8 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & -2 & 1 & 0 \\ 0 & 8 & 0 & -4 & 0 & 1 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 1/4 & 0 \\ 0 & 8 & 0 & -4 & 0 & 1 \end{pmatrix}$$

$$\downarrow$$

$$\left(\underbrace{\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}}_{R} \underbrace{\begin{array}{ccc} 1 & 0 & 0 \\ -.5 & .25 & 0 \\ 0 & -2 & 1 \end{array}}_{B}\right)$$

$R = \text{rref}(A)$

& $BA = R$

(10) $A=\begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix}$

start $\begin{pmatrix} 2 & -3 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{pmatrix}$

$\to \begin{pmatrix} 1 & -3/2 & 1/2 & 0 \\ 4 & 1 & 0 & 1 \end{pmatrix}$

$\to \begin{pmatrix} 1 & -1.5 & .5 & 0 \\ 0 & 7 & -2 & 1 \end{pmatrix}$

$\to \begin{pmatrix} 1 & -1.5 & .5 & 0 \\ 0 & 1 & -\frac{2}{7} & \frac{1}{7} \end{pmatrix}$

$\to \begin{pmatrix} 1 & 0 & (1.5\cdot(-2/7)+.5) & (1.5\cdot 1/7) \\ 0 & 1 & -2/7 & 1/7 \end{pmatrix}$

$\to \begin{pmatrix} 1 & 0 & .07 & .21 \\ 0 & 1 & -.28 & .14 \end{pmatrix}$

(29) $6x_1 - 4x_2 = b_1$

$3x_1 - 2x_2 = b_2$

$\Updownarrow$

$\begin{pmatrix} 6 & -4 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$\Downarrow$

$\begin{vmatrix} 6 & -4 \\ 3 & -2 \end{vmatrix} = -12 - (-12) = 0$

So these two equations are multiples.

$\Rightarrow b_1 = 2b_2$

(30) $\begin{pmatrix} 1 & -2 & 5 & b_1 \\ 4 & -5 & 8 & b_2 \\ -3 & 3 & -3 & b_3 \end{pmatrix} = M$

$REF(M) = \begin{pmatrix} 1 & -2 & 5 & b_2 \\ 0 & 1 & -4 & -b_1 - b_3/3 \\ 0 & 0 & 0 & b_2 - b_1 + b_3 \end{pmatrix}$

$\Rightarrow$

$0 = b_2 - b_1 + b_3$

$\Rightarrow b_1 = b_2 + b_3$!

(32) $M = \begin{pmatrix} 1 & -1 & 3 & 2 & b_1 \\ -2 & 1 & 5 & 1 & b_2 \\ -3 & 2 & 2 & -1 & b_3 \\ 4 & -3 & 1 & 3 & b_4 \end{pmatrix}$

$REF(M) = \begin{pmatrix} 1 & -1 & 3 & 2 & b_1 \\ 0 & -1 & 11 & 5 & 2b_1 + b_2 \\ 0 & 0 & 0 & 0 & b_1 - b_2 + b_3 \\ 0 & 0 & 0 & 0 & b_3 - b_1 + b_4 \end{pmatrix}$

$b_1 = b_2 - b_3$

$\Rightarrow b_1 = b_3 + b_4$

So

$b_2 - b_3 = b_3 + b_4 = b_1$!