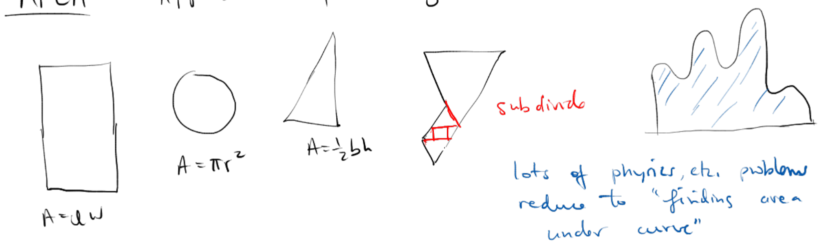
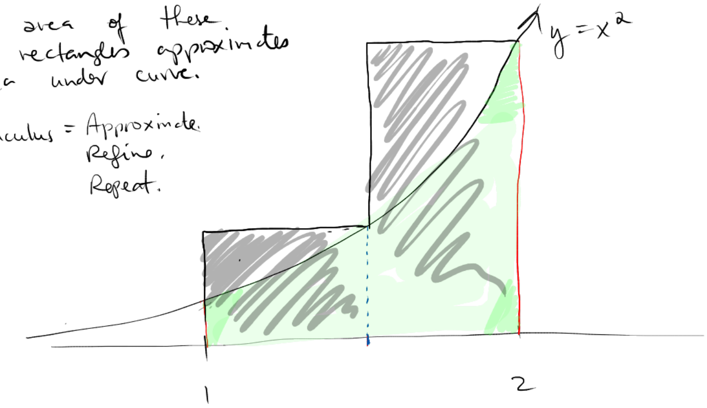
AREA: Application of Integration ___



lots of physics, etc. problems reduce to "finding area under curve"

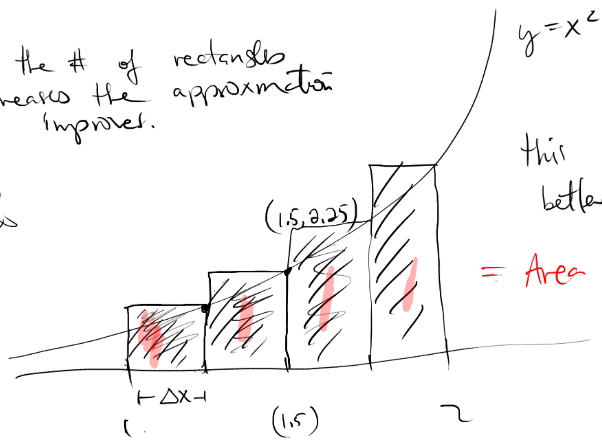
The area of these two rectangles approximates area under curve.

Calculus = Approximate. Refine. Repeat.



as the # of rectangles increases the approximation improves.

$\lim_{n \to \infty}$



this is a better approximation

= Area = sum of 4 rectangles

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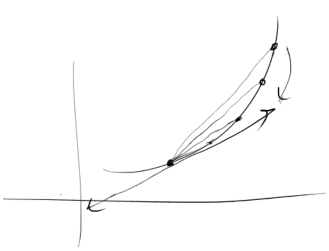
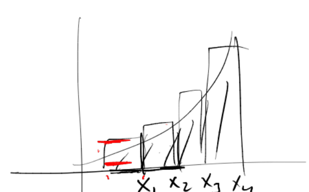
derivative





Area under the curve is a limit of areas of rectangles.

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} \text{height} \cdot \text{width} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)\,dx$$

height: $f(x_i)$ width: Δx

input

sum from 1 to n

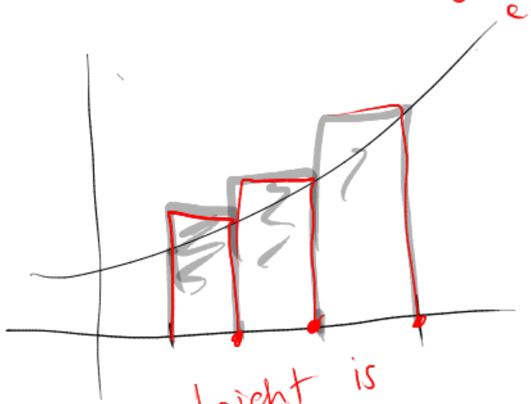
what does $\int_a^b f(x)\,dx$ mean ?

Ex. $\int_1^2 x^2\,dx = \frac{x^3}{3}\Big|_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

This is how to precisely compute area.

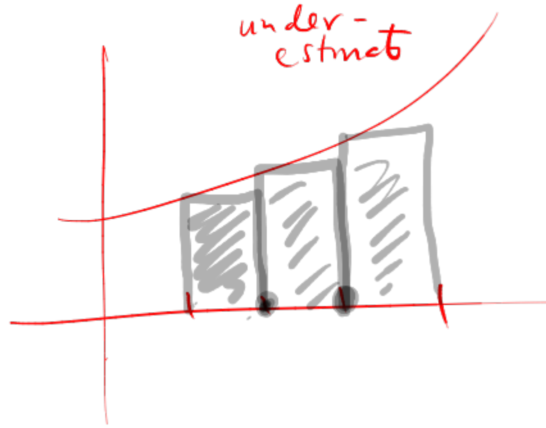
To approximate

over-estimate

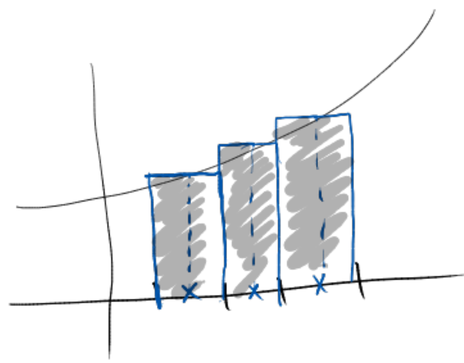


height is based on RIGHT endpoint

under-estimate



LEFT



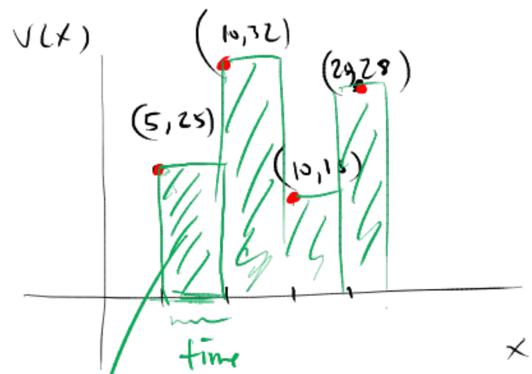
midpoint

when $n \to \infty$
# of rectangles increase but their width decrease $\longrightarrow$ so in the limit it doesn't matter which point you choose

x (time)	5	10	15	20
v(x) (m/h)	25	32	18	28

dist = rate · time

How far did we go



$$\text{area} = 25(10-5) = 125$$

$$= 32(5)$$

$$= 18(5)$$

$$= 20(5)$$

$$= 5(95) = 475$$