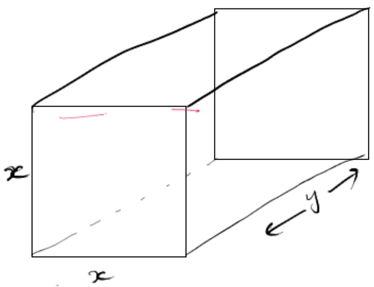


Postal Service Regulates : the girth

↳ "perimeter of smallest side"

Ex: GIRTH $= 4x$, Length $= y$

Requirement: GIRTH + LENGTH ≤ 108 in $\Rightarrow$ $\boxed{4x + y = 108}$

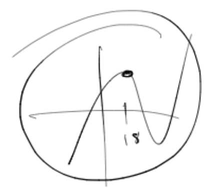
$\Downarrow$

$y = 108 - 4x$

Q: what are the dimensions of the box with the largest volume we can send?

eg what's the largest volume?

$$V = x^2 y$$



$$V(x) = x^2(108 - 4x)$$

$$V(x) = 108x^2 - 4x^3$$

$$V'(x) = 216x - 12x^2 = 0$$

$$x(216 - 12x) = 0$$

$\boxed{x = 0} \longrightarrow$ N/A.

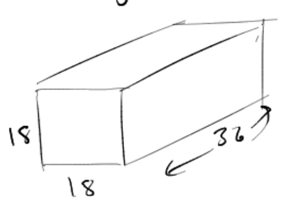
$$216 - 12x = 0$$

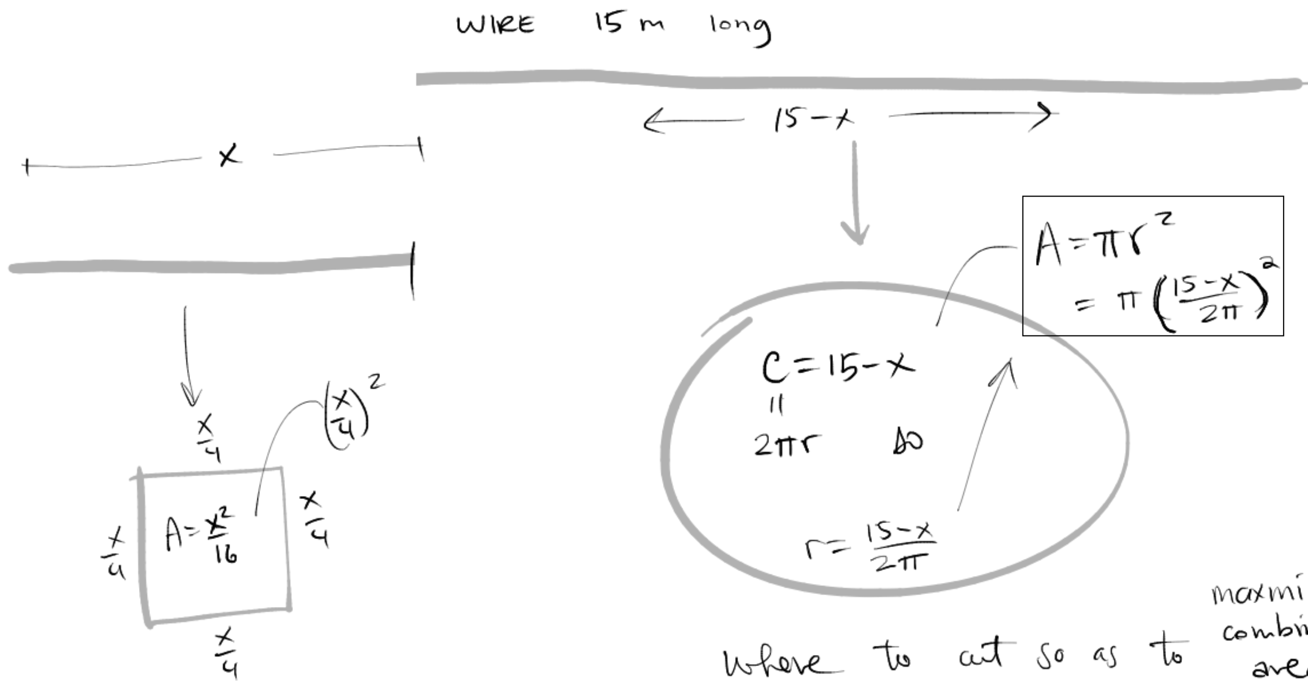
$$216 = 12x \Rightarrow \boxed{x = 18}$$

this is the width giving largest shippable area.

$4x + y = 108$

$x = 18 \quad y = 108 - 72$





Where to cut so as to maximize combined area.

Goal: Maximize $\underbrace{\text{Area of } \square}$ + Area of $\bigcirc$

$$A(x) = \frac{x^2}{16} + \pi\left(\frac{15-x}{2\pi}\right)^2$$

$$A'(x) = \frac{2x}{16} + \boxed{\pi \cdot 2}\left(\frac{15-x}{2\pi}\right)^1 \cdot \left(\frac{-1}{\boxed{2\pi}}\right)$$

$$\left(\frac{15-x}{2\pi}\right)' = \left(\frac{15}{2\pi} - \frac{x}{2\pi}\right)' = 0 - \frac{1}{2\pi} = \frac{-1}{2\pi}$$

$$A'(x) = 16\pi\left[\frac{x}{8} - \left(\frac{15-x}{2\pi}\right)\right] = (0)16\pi$$

$$2\pi x - 8(15-x) = 0$$

$$8x + 2\pi x - 120 = 0$$

$$x(8+2\pi) - 120 = 0$$

$$x = \frac{120}{8+2\pi} = 8.4 \text{ m}$$

$$A(8.4) = \frac{(8.4)^2}{16} + \pi\left(\frac{15-8.4}{2\pi}\right)^2$$