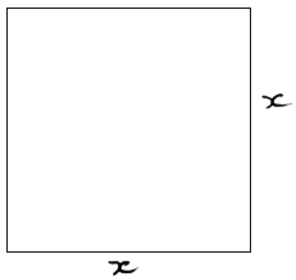
How does the rate of change of the AREA depend on the rate of change of the SIDE LENGTH?



$A = (x(t))^2$, rate of change of the area with respect to time is

$\frac{dA}{dt} = \frac{d}{dt}(x(t))^2 = 2(x(t))^{1} \cdot \frac{dx}{dt}$

chain rule

derivative of A wrt. time

$A = x^2$ think x is function of time

$$\boxed{\frac{dA}{dt} = 2x\frac{dx}{dt}}$$

Question: What is the $\underbrace{\text{rate of change of the area of a square}}_{= \frac{dA}{dt}}$ when the $\underbrace{\text{side length is 5}}_{x=5}$ and the $\underbrace{\text{lengths are growing at 3 in/sec}}_{\frac{dx}{dt}=3}$?

$\frac{dx}{dt} = 3 \Rightarrow \frac{dA}{dt} = 2(5)\cdot 3 = 30 \ \frac{in^2}{sec}$

Next, ... cubes -

What is the rate of change of the VOLUME OF A CUBE when its $\overbrace{\text{side length is 5}}^{x=5}$ and the lengths are growing at 3 in/s? $\frac{dx}{dt} = 3$

comes from Volume: $V = x^3$

what do we want: $\frac{dV}{dt}$

$$\boxed{\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}}$$

$$\frac{dV}{dt} = 3(5)^2 \cdot 3 = 225 \ \frac{in^3}{s}$$

What is the rate of change of the VOLUME OF A CUBE when its side length is 10 and the lengths are growing at 1 in/s?

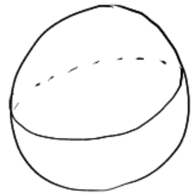
$$\frac{dV}{dt} = 3(10)^2 \cdot 1 = 300 \ \frac{in^3}{s}$$

3-D quantity

$V = \frac{4}{3}\pi r^{3}$

radius

round



surface area

$SA = 4\pi r^2$

$\frac{dV}{dr} = 3 \cdot \frac{4}{3}\pi r^2 = 4\pi r^2$

$dV/dt \leftarrow$ build

Suppose a spherical balloon is being inflated at a constant rate of 20 cubic inches per second. At what rate is its radius increasing when the diameter of the sphere is 12 inches?

dr/dt

$d = 2r$

$12 = 2r$

$r = 6$

$V = \frac{4}{3}\pi r^3$

d/dt

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$

I know

$20 = 4\pi(6)^2 \cdot \frac{dr}{dt} = \quad dr = \frac{20}{4\pi \cdot 36} = \boxed{.0442}$

Does the rate of change of the radius increase or decrease as the radius increases? What is dr/dt when r = 24?

$r = 24$

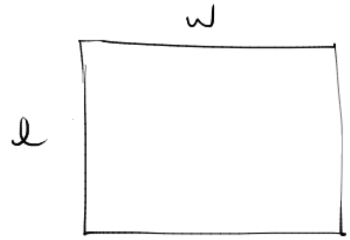
$20 = 4\pi(24)^2 \cdot \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{20}{4\pi(24)^2} = \boxed{.0027}$

$$\text{Position} \xrightarrow{d/dt} \text{Velocity}$$

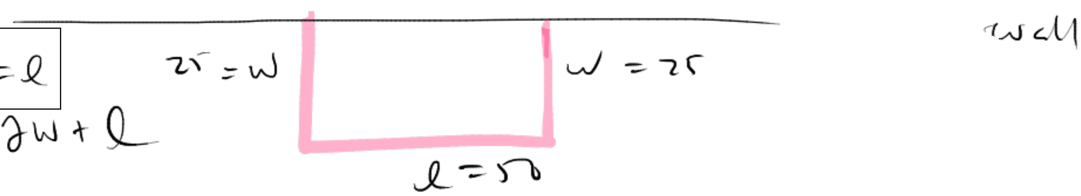
$y = 35 + \sin(2\pi x)$ $\qquad$ $y' = \cos(2\pi x) \cdot 2\pi$

100

$2l + 2w = 100 \Rightarrow 2w = 100 - 2l \qquad w = 50 - l$



$$A = l \cdot w$$
$$= l(50 - l)$$
$$A = 50l - l^2$$
$$A' = 50 - 2l = 0$$
$$50 = 2l$$
$$l = 25$$



$\boxed{100 - 2w = l}$

$100 = 2w + l$

$$A = l \cdot w = (100 - 2w)w = 100w - 2w^2$$

$$A' = 100 - 4w = 0$$

$$100 = 4w \Rightarrow \frac{100}{4} = w = 25$$