

Exam \*should\* be out of 60. With a generous curve, it's out of 80.

Name

Exam 4 :: Math 271 :: April 26, 2017

INSTRUCTIONS: Choose any seven of the eight problems.

1. Ryan fires a projectile from the ground (height 0) upward at an initial velocity of  $480 \frac{ft}{s}$ . Assuming the effect of air resistance is negligible and the effect of gravity is  $-32 \frac{ft}{sec}$ . Answer the following questions.

$$v(0) = 480, \quad a(t) = -32$$

$$v(t) = \int a(t) dt = \int -32 dt = -32t + C$$

$= -32(0) + 480$

$v(0)$

$$v(t) = -32t + 480$$

$$s(t) = \int v(t) dt = \int -32t + 480 dt = -\frac{32t^2}{2} + 480t + C$$

$$s(t) = -16t^2 + 480t$$

from the ground,  
 $s(0) = 0$   
"C"

When does the projectile strike the ground?

solve  $s(t) = 0$

$$-16t^2 + 480t = 0$$

$$t^2 - 30t = 0$$

$$t(t-30) = 0$$

$$t = 0$$

$$t = 30$$

With what speed does the projectile strike the ground?

$$v(30) = -32(30) + 480 = -480$$

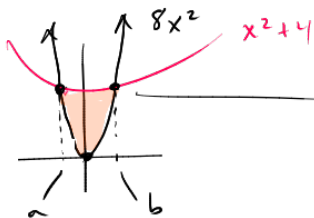
2. Find the average velocity of a projectile whose velocity in ( $\frac{ft}{sec}$ ) is given by  $v(t) = t^3$  where  $t$  is measured in seconds and  $t \in [0, 5]$ .

Avg  
Value

$$\frac{1}{b-a} \int_a^b f(t) dt$$

$$\text{Avg Velocity} = \frac{1}{5-0} \int_0^5 t^3 dt = \frac{1}{4.5} t^4 = \frac{1}{20} t^4 \Big|_0^5 = \frac{5^4}{20} \approx 31.1$$

3. Find the area bound by  $y = 8x^2$  and  $y = x^2 + 4$ . Sketch the region.



$$\text{set } 8x^2 = x^2 + 4$$

$$7x^2 = 4$$

$$x^2 = \frac{4}{7} = \pm \frac{2}{\sqrt{7}}$$

$$\int_a^b x^2 + 4 - 8x^2 dx = \int_{-\frac{2}{\sqrt{7}}}^{\frac{2}{\sqrt{7}}} 4 - 7x^2 dx = 4x - \frac{7x^3}{3} \Big|_{-\frac{2}{\sqrt{7}}}^{\frac{2}{\sqrt{7}}}$$

Even.  $4 - 7x^2 \Rightarrow$  <sup>double</sup>  $2 \left( 4x - \frac{7x^3}{3} \right) \Big|_0^{\frac{2}{\sqrt{7}}} = 2 \left( 4 \left( \frac{2}{\sqrt{7}} \right) - \frac{7 \left( \frac{2}{\sqrt{7}} \right)^3}{3} \right)$

area = right. =  $\boxed{4.01}$

4. Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2, y = 0, x = 0, x = 2$$

about the y-axis.

- Integrate w.r.t  $y$
- $y = x^2 \rightarrow x = \sqrt{y}$

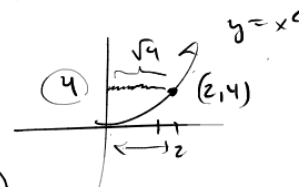
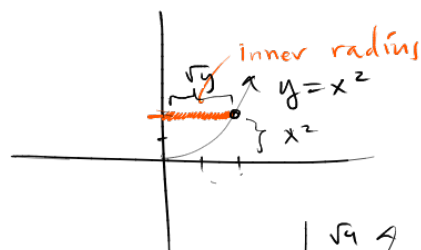
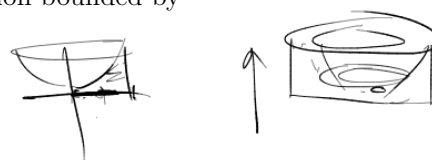
$$\text{Big} - \text{Small}$$

$$\pi \cdot 2^2 - \pi (\sqrt{y})^2$$

$$4\pi - y\pi = \pi(4 - y)$$

$$\int_0^4 \pi(4 - y) dy = \pi \left( 4y - \frac{y^2}{2} \right) \Big|_0^4$$

$$= \pi(16 - 8) = 8\pi$$



5. Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2, y = 0, x = 0, x = 2$$

about the x-axis.