Differentiation - Power Rule

Find $f'(x)$.

1. $f(x) = x^4 + x^3 + x^2 + x + 1 \Longrightarrow f'(x) = 4x^3 + 3x^2 + 2x + 1$

2. $f(x) = 7x^2 - 2x^3 + 3 \Longrightarrow f'(x) = 14x - 6x^2$

3. $f(x) = \dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3} = x^{-1} + x^{-2} + x^{-3}$

$$\Longrightarrow f'(x) = -x^{-2} - 2x^{-3} - 3x^{-4} = -\dfrac{1}{x^2} - \dfrac{2}{x^3} - \dfrac{3}{x^4}$$

4. $f(x) = \dfrac{2}{x} - \dfrac{1}{3x^2} + \dfrac{4}{7x^3} = 2x^{-1} - \dfrac{1}{3}x^{-2} + \dfrac{4}{7}x^{-3}$

$$\Longrightarrow f'(x) = -2x^{-2} - \dfrac{1}{3}(-2)x^{-3} + \dfrac{4}{7}(-3)x^{-4} = -\dfrac{2}{x^2} + \dfrac{2}{3x^3} - \dfrac{12}{7x^4}$$

5. $f(x) = \sqrt{x} + x\sqrt{x} + x^2\sqrt{x} = x^{1/2} + x^{3/2} + x^{5/2}$

$$\Longrightarrow f'(x) = \dfrac{1}{2}x^{-1/2} + \dfrac{3}{2}x^{1/2} + \dfrac{5}{2}x^{3/2} = \dfrac{1}{2\sqrt{x}} + \dfrac{3}{2}\sqrt{x} + \dfrac{5}{2}x\sqrt{x}$$

6. $f(x) = 5\sqrt{x} - 8x^2\sqrt{x} + \pi - 4x = 5x^{1/2} - 8x^{5/2} + \pi - 4x$

$$\implies f'(x) = 5\left(\frac{1}{2}\right)x^{-1/2} - 8\left(\frac{5}{2}\right)x^{3/2} + 0 - 4 = \frac{5}{2\sqrt{x}} - 20x\sqrt{x} - 4$$

7. $f(x) = \dfrac{2}{\sqrt{x}} + \dfrac{3}{x\sqrt{x}} + \dfrac{4}{x^2\sqrt{x}} + \dfrac{5}{7} = 2x^{-1/2} + 3x^{-3/2} + 4x^{-5/2} + \dfrac{5}{7}$

$$\implies f'(x) = -x^{-3/2} - \frac{9}{2}x^{-5/2} - 10x^{-7/2} + 0 = -\frac{1}{x\sqrt{x}} - \frac{9}{2x^2\sqrt{x}} - \frac{10}{x^3\sqrt{x}}$$

8. $f(x) = \dfrac{3}{2\sqrt{x}} + \dfrac{1}{\sqrt[3]{x}} - \dfrac{4}{3\sqrt[4]{x}} = \dfrac{3}{2}x^{-1/2} + x^{-1/3} - \dfrac{4}{3}x^{-1/4}$

$$\implies f'(x) = -\frac{3}{4}x^{-3/2} - \frac{1}{3}x^{-4/3} + \frac{1}{3}x^{-5/4} = -\frac{3}{4x\sqrt{x}} - \frac{1}{3x\sqrt[3]{x}} + \frac{1}{3x\sqrt[4]{x}}$$

9. $f(x) = x\ln 3 + \dfrac{1}{2}x\sqrt[3]{x} - x^2\sqrt[4]{x} = x\ln 3 + \dfrac{1}{2}x^{4/3} - x^{9/4}$

$$\implies f'(x) = \ln 3 + \frac{2}{3}x^{1/3} - \frac{9}{4}x^{5/4} = \ln 3 + \frac{2}{3}\sqrt[3]{x} - \frac{9}{4}x\sqrt[4]{x}$$

10. $f(x) = 3x\sqrt[3]{x} - 7x + \dfrac{4}{\sqrt{x}} - 5e = 3x^{1/3} - 7x + 4x^{-1/2} - 5e$

$$\implies f'(x) = x^{-2/3} - 7 - 2x^{-3/2} - 0 = \frac{1}{\sqrt[3]{x^2}} - 7 - \frac{2}{x\sqrt{x}}$$