10/13/16
Name

Antiderivatives 2

Find the antiderivative (#'s 1 - 5).

$$1.\ \int 6\sqrt{x} - \sqrt[6]{x}\,dx = \int 6x^{1/2} - x^{1/6}\,dx = 6\cdot\frac{x^{3/2}}{3/2} - \frac{x^{7/6}}{7/6} + C$$
$$= 4x^{3/2} - \frac{6}{7}x^{7/6} + C = 4x\sqrt{x} - \frac{6}{7}x\sqrt[6]{x} + C$$

$$2.\ \int \frac{5-4x^2+3x^4}{x^3}\,dx = \int 5x^{-3} - 4\left(\frac{1}{x}\right) + 3x\,dx = 5\cdot\frac{x^{-2}}{-2} - 4\ln|x| + 3\cdot\frac{x^2}{2} + C$$
$$= -\frac{5}{2x^2} - 4\ln|x| + \frac{3}{2}x^2 + C$$

$$3.\ \int \frac{3}{x\sqrt{x^2-1}} + 7e^x\,dx = \int 3\left(\frac{1}{x\sqrt{x^2-1}}\right) + 7e^x\,dx = 3\sec^{-1}x + 7e^x + C$$

$$4.\ \int 2\sec x\tan x + 3\sin x\,dx = 2\sec x + 3(-\cos x) + C = 2\sec x - 3\cos x + C$$

$$5.\ \int \sqrt{5}\csc^2 x + \sqrt{2}\,dx = \sqrt{5}(-\cot x) + x\sqrt{2} + C = -\sqrt{5}\cot x + x\sqrt{2} + C$$

Find $f(x)$ (#'s 6 - 7).
6. $f'(x) = 6\sec x\tan x + e^x$, $f(0) = 4$

$$f(x) = \int 6\sec x\tan x + e^x\,dx = 6\sec x + e^x + C$$

$$4 = f(0) = 6\sec(0) + e^0 + C = 7 + C \longrightarrow C = -3$$

Answer: $f(x) = 6\sec x + e^x - 3$

7. $f''(x) = 6x - 8,\ f'(0) = 3,\ f(1) = 5$

$$f'(x) = \int 6x - 8\,dx = 3x^2 - 8x + C$$

$$3 = f'(0) = 3(0)^2 - 8(0) + C \longrightarrow C = 3$$

$$f(x) = \int 3x^2 - 8x + 3\,dx = x^3 - 4x^2 + 3x + C$$

$$5 = f(1) = (1)^3 - 4(1)^2 + 3(1) + C \longrightarrow C = 5$$

Answer: $f(x) = x^3 - 4x^2 + 3x + 5$

Use the suggested substitution to find the antiderivative (#'s 8 - 10). CHECK YOR ANSWER.

8. Use $u = 3x + 4 \rightarrow du = 3\,dx \rightarrow \frac{1}{3}\,du = dx$.

$$\int 3e^{3x+4}\,dx = \int 3e^u \left(\frac{1}{3}\right)\,du = \int e^u\,du = e^u + C = e^{3x+4} + C$$

Check:

$$\frac{d}{dx}\left[e^{3x+4}\right] = \frac{e}{dx}\left[3x+4\right] e^{3x+4} = 3e^{3x+4}$$

9. Use $u = x^2 + 1 \rightarrow du = 2x\,dx \rightarrow \frac{1}{2x}\,du = dx$.

$$\int 2xe^{x^2+1}\,dx = \int 2x\,e^u \left(\frac{1}{2x}\right)\,du = \int e^u\,du = e^u + C = e^{x^2+1} + C$$

Check:

$$\frac{d}{dx}\left[e^{x^2+1}\right] = \frac{d}{dx}\left[x^2+1\right] e^{x^2+1} = 2x\,e^{x^2+1}$$

10. Use $u = x^3 \rightarrow du = 3x^2\,dx \rightarrow \frac{1}{3x^2}\,du = dx$.

$$\int 3x^2 \sec\left(x^3\right) \tan\left(x^3\right)\,dx = \int 3x^2 \sec u \tan u \left(\frac{1}{3x^2}\right)\,du = \int \sec u \tan u\,du$$

$$= \sec u + C = \sec(x^3) + C$$

Check:

$$\frac{d}{dx}\left[\sec\left(x^3\right)\right] = \frac{e}{ex}\left[x^3\right] \sec\left(x^3\right) \tan\left(x^3\right) = 3x^2 \sec\left(x^3\right) \tan\left(x^3\right)$$