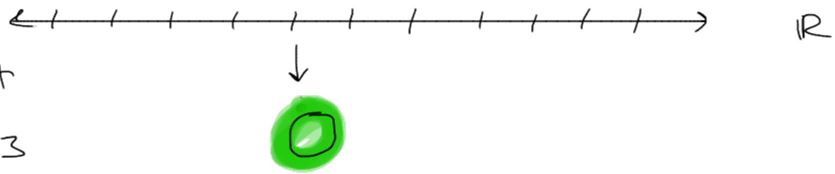
Week 10 – Monday

1-Week from Wednesday

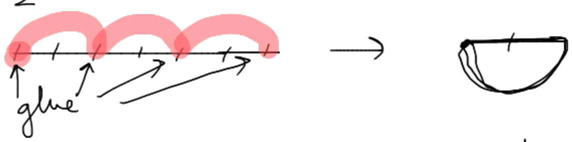
1. Homework Due
2. Topic for Project / Annotated Biblio — what sources / how you will use it

Quotient Space



quotient

$\frac{6}{2} = 3$



Presentations:

- live / synchronous online
- recorded:

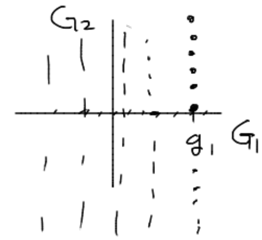
This week:

- I will trim down some of HW assignments.
- Grading is coming but slow

$30 \sim 10^{+} = 300^{+}$ pages

External Direct Products of Groups

similar to $\mathbb{R}^2$. $\mathbb{R}^2 = \underset{\text{variable } x}{\mathbb{R}} \oplus \underset{y}{\mathbb{R}}$ $(x, y) \in \mathbb{R}^2$.



Def'n: let A, B, C be groups w/ $a_i \in A$, $b_j \in B$, $c_k \in C$

$$\underbrace{A \oplus B \oplus C}_{\text{external direct product}} = \{ \underbrace{(a_i, b_j, c_k)} \mid \text{for } a_i, b_j, c_k \text{ above} \}$$

elements are n-tuples ($n = 3$ for this case)

operation: componentwise multiplication

Ex: $\overset{+}{\mathbb{Z}} \oplus \overset{+}{\mathbb{Z}_2} = \{ (n, a) \mid n \in \mathbb{Z}, a \in \mathbb{Z}_2 \}$

ex: $(1, 0)$, $(-5, 1)$, $(3, 1)$ live here.

group operation: (component-wise addition w/ modulo 2 in 2^{nd} coord).

ex: $(1, 0) + (-5, 1) = (-4, 1)$

$(-5, 1) + (3, 1) = (-2, 2 \bmod 2) = (-2, 0)$

ex: $e \in \mathbb{Z} \oplus \mathbb{Z}_2$ is $e = (0, 0)$

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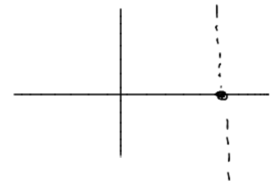
what's the id element in $A \oplus B \oplus C$?

(e_A, e_B, e_C)

Ex. $\underbrace{U(3)}_{\substack{\{1,2\} = \mathbb{Z}_2 \\ \text{mod } 3}} \oplus \underbrace{U(4)}_{\substack{\{1,3\} = \mathbb{Z}_2 \\ \text{mod } 4}} = \{(1,1), \underbrace{(1,3)}_{\text{order } 2}, \underbrace{(2,1)}_{\text{order } 2}, \underbrace{(2,3)}_{\text{order } 2}\}$



$|U(3) \oplus U(4)| = |U(3)| \times |U(4)| = 4$

what group is it isomorphic to? Cyclic? $\langle (2,3) \rangle = \{(2,3), (1,1)$

- $(2,3)^2 = (2,3)\cdot(2,3) = (4 \bmod 3,\ 9 \bmod 4) = (1,1)$ (identity)
- $(2,3)^3 = (2,3)^2 \cdot (2,3) = (2,3) \implies |(2,3)| = 2$
- $(2,1)^2 = (4 \bmod 3, 1) = (1,1) \qquad |(2,1)| = 2$
- $(1,3)^2 = (1, 9 \bmod 4) = (1,1) \qquad |(1,3)| = 2$

$U(3) \oplus U(4) \cong \{H, V, R_{180}, R_0\} \le D_4$

$\cong \mathbb{Z}_2 \oplus \mathbb{Z}_2 \cong V_4$

Fact: There are exactly 2 groups of order 4 (up to isomorphism)

$\mathbb{Z}_4, \quad \mathbb{Z}_2 \oplus \mathbb{Z}_2$