Thursday – Week 10

warm-up

$$\int e^x \sin(e^x)\,dx$$

$e^x \longrightarrow e^x$

$\frac{d}{dx}(-\cos(e^x)+c)$ → 0

$-(-\sin(e^x)\cdot e^x)$ → $\frac{d}{dx}(e^x)$

$= \sin(e^x)\cdot e^x$ ✓

$$= \int \sin(u)\,du$$

$u = e^x$

$du = \underbrace{e^x\,dx}$

$$\int \sin(e^x)\cdot \underbrace{e^x\,dx}$$

$$= \int \sin(u)\,du = -\cos(u)+c = \boxed{-\cos(e^x)+c}$$

---

warm-up #2

Think: I see degree 1 diff (derivative relationship)

$u = x^2+1$

$du = \underbrace{2x\,dx}$

$$\int 2x(x^2+1)^4\,dx$$

$$= \int \underbrace{(x^2+1)}_{u}{}^4\,\underbrace{2x\,dx}_{du} = \int u^4\,du = \frac{u^5}{5}+c = \boxed{\frac{(x^2+1)^5}{5}+c}$$

$du/u$ Anti-derivatives $\longrightarrow$ $\int \frac{du}{u} = \ln|u| + c$

$$\int \frac{3x^2}{(5x^3+1)}\,dx = \int \frac{\cancel{3x^2}}{u}\,\frac{1}{15\cancel{x^2}}\,du = \frac{1}{5}\int \frac{1}{u}\,du = \boxed{\frac{1}{5}\ln|5x^3+1| + c}$$

see: $5x^3+1$ higher degree

$\boxed{u = 5x^3+1}$ | $\boxed{dx = \frac{1}{15x^2}\,du}$

$du = 15x^2\,dx$

$$\int \frac{5e^x}{e^x+1}\,dx = \int \frac{5\cancel{e^x}}{u}\,\frac{1}{\cancel{e^x}}\,du = 5\int \frac{1}{u}\,du = \boxed{5\ln|e^x+1| + c}$$

$\boxed{u = e^x+1}$ | $\boxed{dx = \frac{1}{e^x}\,du}$

$du = e^x\,dx$

**Antiderivatives 5**

Find the indicated antiderivative. Check your answers.

1. $\int \frac{2x}{x^2+1}\,dx =$ $\quad \ln|x^2+1| + c$

2. $\int \frac{\cos x}{1+\sin x}\,dx =$ $\quad \int \frac{du}{u} = \ln|1+\sin x| + c$

$u = 1 + \sin x$

$du = \cos x\,dx$

3. $\frac{1}{4}\int \frac{4x^3}{1+x^4}\,dx =$ $\quad \frac{1}{4}\int \frac{du}{u} = \frac{1}{4}\ln|1+x^4| + c$

Think: degree 1 difference!

$u = 1+x^4$

$du = 4x^3\,dx$

4. $\int \frac{e^x+2}{e^x+2x+7}\,dx =$ $\quad \ln|e^x+2x+7| + c$

Think: derivative relationship

$u = e^x + 2x + 7$

$du = e^x + 2\,dx$

5. $\int \frac{3}{2x+5}\,dx =$ $\quad \frac{3}{2}\int \frac{2\cdot 1}{2x+5}\,dx = \frac{3}{2}\int \frac{du}{u} = \boxed{\frac{3}{2}\ln|2x+5| + c}$

$u = 2x+5$

$du = 2\,dx$

6. $\displaystyle\int \frac{2x}{(x^2+1)^2}\,dx =$

$n$ = negative

$\int u^n \, du$

7. $\displaystyle\int \frac{\cos x}{(1+\sin x)^2}\,dx =$

8. $\displaystyle\int \frac{x^3}{(1+x^4)^3}\,dx =$

9. $\displaystyle\int \frac{e^x+2}{(e^x+2x+7)^3}\,dx =$

10. $\displaystyle\int \frac{3}{(2x+5)^4}\,dx =$

# Inverse Trig Anti-D's

Common Forms -

$$\int \frac{1}{1+x^2}\,dx$$

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx$$

$$\boxed{\int \frac{1}{\sqrt{1-x^2}}\,dx} = \sin^{-1}(x)$$

$$\boxed{\begin{aligned} \int \frac{1}{1+u^2}\,du &= \tan^{-1}u \\ \int \frac{1}{u\sqrt{u^2-1}}\,du &= \sec^{-1}u \\ \int \frac{1}{\sqrt{1-u^2}}\,du &= \sin^{-1}u \end{aligned}}$$

$u = f(x)$ (say $x^2$ or $e^x$)

$du = f'(x)\,dx$ (or $2x\,dx$ or $e^x\,dx$)

Ex. suppose: $f(x) = \sin^{-1}(x^2)$

$u = x^2 \rightarrow \frac{du}{dx} = 2x$

$$f'(x) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

So

$$\int \frac{2x}{\sqrt{1-x^4}}\,dx =$$

Think: do I see degree 1 diffs? - NO

($u = 1-x^4$, $\underbrace{du = -4x^3\,dx}_{\text{dead end.}}$)

recognize $\sqrt{\quad}$ & $1-(x)^{\text{even power}}$

now think $\sin^{-1}(u)$ ...

$u = \underline{x^{1/2(\text{even power})}}$ (sqrt)

In this case $u = \sqrt{x^4} = x^2$

$$\int \frac{2x}{\sqrt{1-x^4}}\,dx \overset{\text{sub}}{=} \int \frac{\cancel{2x}}{\sqrt{1-u^2}} \frac{1}{\cancel{2x}}\,du = \int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}(u) + C$$

$x^4 \to u^2$?

set $\boxed{u = x^2}$ square $\Rightarrow u^2 = x^4$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$$\boxed{\frac{1}{2x}\,du = dx}$$

$$\boxed{\sin^{-1}(x^2) + C}$$