Wed. Week 12

Factor groups (quotienting out by normal subgroup) causes systematic collapse in the group.

Ex $G = D_4$

$K = \{r_0, r_{180}\} = r_{180}K$

cosets of K

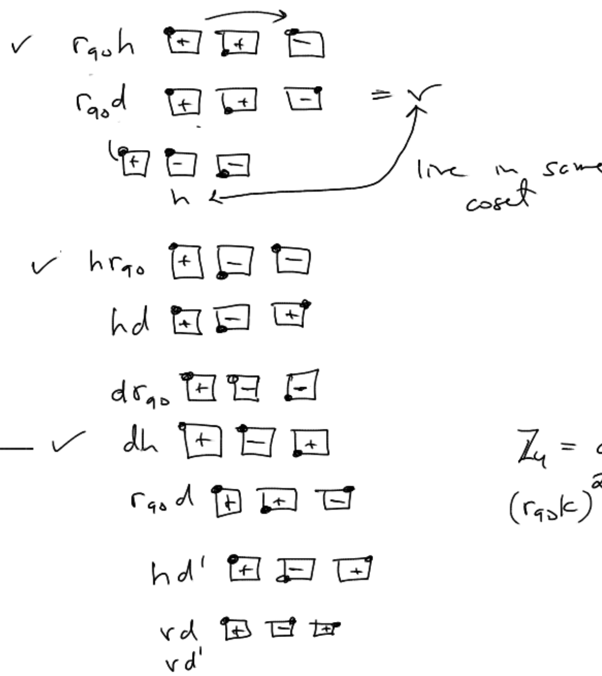
$r_{90}K = \{r_{90}, r_{270}\}$, $hK = \{h, hr_{180}\} = \{h, v\} = vK$, $dK = \{d, d'\}$

Cayley Table for D_4/K

	K	$r_{90}K$	hK	dK
K	K	$r_{90}K$	hK	dK
$r_{90}K$	$r_{90}K$	K	dK	hK
hK	hK	dK	K	$r_{90}K$
dK	dK	hK	r_{90}	K

↕

	r_0	r_{180}	r_{90}	r_{270}	h	v	d	d'
r_0	r_0	r_{180}	r_{90}	r_{270}	h	v	d	d'
r_{180}	r_{180}	r_0	r_{270}	r_{90}	v	h	d'	d
r_{90}	r_{90}	r_{270}	r_{180}	r_0	d	d'	v	h
r_{270}	r_{270}	90	r_0	r_{180}	d'	d	h	v
h	h	v	d	d'	r_0	r_{180}	r_{90}	r_{270}
v	v	h	d'	d	r_{180}	r_0	r_{270}	r_{90}
d	d	d'	h	v	r_{90}	r_{270}	r_0	r_{180}
d'	d'	d	v	h	r_{270}	r_{90}	r_{180}	r_0



$\mathbb{Z}_4$ = cyclic

$(r_{90}K)^2 = K$

$D_4/K \cong V_4$

$|D_4/K| = |D_4 : K| = \frac{|D_4|}{|K|} = \frac{8}{2} = 4$

Ex: $G = \mathbb{Z}_4 \oplus \mathbb{Z}_4$

$H = \{(0,0), (2,0), (0,2), (2,2)\}$. Claim $H \trianglelefteq G$ (don't prove it)

What group is G/H isomorphic to ? ($\mathbb{Z}_4$ or V_4)

$|G/H| = |G|/|H| = 16/4 = 4$

Q: Is there an elt. of order 4 ? (Is every elt. of order 2).

Let $(a,b) \in G$. ($a \in \{0,1,2,3\}$, $b \in \{0,1,2,3\}$)

the coset:

$(a,b) + H = \{(a,b), (2+a, b), (a, 2+b), (2+a, 2+b)\}$.

$$((a,b)+H)^2 = ((a,b)+H) + ((a,b)+H) = (2a, 2b) + H.$$

if $a=1, b=1$ this becomes $(2,2) + H = H$ (order 2)

if $a=0, b=1$: $(0,2)+H = H$ (H absorbs its elts) (order 2)

$a=1, b=0$: $(2,0) + H = H$ (order 2)

$a=0, b=0$ $(0,0)+H = H$ (order 2)

if $a=2, b=3$: $(4,6) + H \overset{\%4}{=} (0,2) + H = H$ (order 2)

$a=1, b=3$: $(2,6) + H \overset{\%4}{=} (2,2) + H = H$ order 2

⋮ $\Rightarrow$ every elt. of order 2 $\Rightarrow$ V_4

Now let $K = \{(1,2), (2,0), (3,2), (0,0)\}$, $K \trianglelefteq \mathbb{Z}_4 \oplus \mathbb{Z}_4$ ← abelian, every subgrp is normal.

K absorbs these elts.

$|K| = 4 \Rightarrow G/K \approx \mathbb{Z}_4$ or V_4.

Look for generator of order 4.

$((1,1) + K)^4 = (4,4) + K \overset{\%4}{=} (0,0) + K = K$ ((1,1) might be of order 4)

$\langle (1,1) + K \rangle = \{(1,1) + K, (2,2) + K, (3,3) + K, (0,0) + K\}$

- $(1,1)+K$: (2,3), (3,1), (0,3), (1,1)
- $(2,2)+K$: (3,0), (0,2), (1,0), (2,2)
- $(3,3)+K$: (0,1), (1,3), (2,1), (3,3)
- $(0,0)+K$ = K

cosets partition $\Rightarrow$ no two cosets are equal

$|(1,1)+K| = 4$ so $\mathbb{Z}_4 \oplus \mathbb{Z}_4 / K \approx \mathbb{Z}_4$