

① fix pre-image of \mathbb{Z}_2

② 1st Isom thm:

- D_4

- \mathbb{Z}_4

- $A_4 \trianglelefteq S_4$ so $f: (x) = \begin{cases} 0 & \text{even} \\ 1 & \text{odd} \end{cases}$ has $\ker A_4 \mid \text{im}(f) \cong \mathbb{Z}_2$

- Index 2 subgroups normal

Let $A = A_4$, $x \notin A$.

$xA \neq A$ | Two cosets: xA, A equal
 $Ax \neq A$ | Ax, A equal
 \therefore equal

- Wrapping function

③ Homomorphisms give some properties

- size of kernel reflects similarity

Isom. give alternate view of whole group

$D_4 / \langle \text{elements} \rangle \cong \mathbb{Z}_4$ (closely related)
vs $S_4 / A_4 \cong \mathbb{Z}_2$ (not $\cong S_4$ (large kernel))
(people study all possible homom. (G))