$f(e) = e'$

suppose $f(e) = x$ and $f(g) = g'$

f is a homomorphism so $g' = f(g) = f(e.g) = f(e).f(g) = x.g'$

then apply the inverse of g' on the right to both sides:

$e = x$

Homomomorphisms $\qquad \phi : G \to \bar{G}$

1. $\phi(e) = \bar{e}$ b/c $\quad \phi(e) = \bar{g}$ for some $\bar{g} \in \bar{G}$ & $\phi(g) = \phi(g \cdot e) = \phi(g) \cdot \phi(e)$

$\Rightarrow$ apply inverses: $\underbrace{(\phi(g))^{-1}(\phi(g))}_{\bar{e}} = \underbrace{(\phi(g))^{-1}\phi(g)}_{\bar{e}}\ \phi(e)$

$\bar{e} = \phi(e)$

---

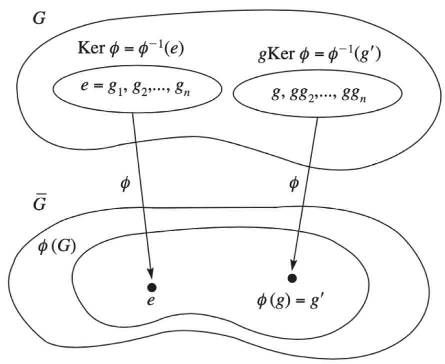
2. $\phi(a) = \phi(b) \Leftrightarrow a\,\mathrm{Ker}(\phi) = b\,\mathrm{Ker}(\phi)$ (or $a = bk$ for some $k \in \mathrm{Ker}$)

($\Rightarrow$) Assume $\phi(a) = \phi(b)$. Let $x \in a\,\mathrm{Ker}(\phi)$, $x = ak$ for $k \in \mathrm{ker}(\phi)$

$\phi(ab^{-1}) = \phi(a)\phi(b^{-1}) = \phi(b)\phi(b^{-1}) = \phi(e) = \bar{e} \Rightarrow ab^{-1} \in \mathrm{Ker} \Rightarrow ab^{-1} = k$ $\boxed{a = bk}$

($\Leftarrow$) If $a\,\mathrm{Ker}(\phi) = b\,\mathrm{Ker}(\phi)$ then $a = bk$ for some $k \in \mathrm{Ker}(\phi)$.

then $\phi(a) = \phi(bk) = \phi(b)\phi(k) = \phi(b)\bar{e} = \phi(b)$ ✓



Ex. $f : \mathbb{C}^* \to \mathbb{C}^*$ by $f(z) = z^4$.

$K = \{1, -1, i, -i\}$

$\varphi(\sqrt{2}) = (\sqrt{2})^4 = 2$

what else hits 2?

$\sqrt{2}\,\mathrm{Ker}(f) = \sqrt{2}K = \{\sqrt{2}, -\sqrt{2}, i\sqrt{2}, -i\sqrt{2}\}$

Ex $f : S_3 \to \mathbb{Z}_2$ $\quad f(x) = 0$ if x even, $f(x) = 1$ if x odd.

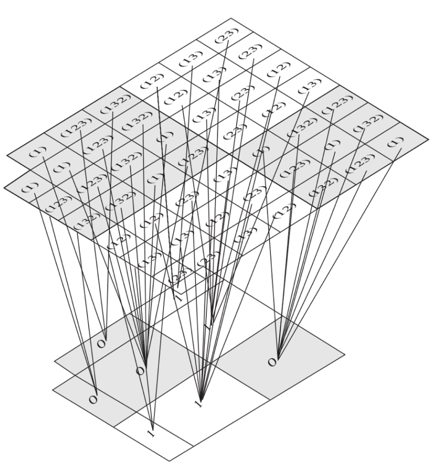


Figure 10.2 Homomorphism from S_3 to Z_2.

1st Isom. theorems:

$\phi: G \to \bar{G}$

$G/_{\ker(\phi)} \cong \phi(G)$

— Map —

$\psi: (g\ker(\phi)) = \phi(g)$

Well-def:

$$\begin{array}{ccc} a\ker & = & b\ker \\ \psi \downarrow & & \downarrow \psi \\ \phi(a) & & \phi(b) \end{array}$$

$\Leftrightarrow a\ker = b\ker$

1-1

If

$\psi(a\ker) = \psi(b\ker)$

$\Rightarrow \phi(a) = \phi(b)$

$\Rightarrow a\ker = b\ker$

Ex. $\phi: D_4 \to D_4$

$$\begin{array}{cccc} & & d, d' & \\ (r_0, & r_{180}, & v, & h) \\ \uparrow & \uparrow & & \uparrow \\ (0,180) & h, v & & 90, 270 \end{array}$$

Ex $\mathbb{Z}/_{n}\mathbb{Z} \cong \mathbb{Z}_n$

Wrapping Function

Circle Group : $H = \{a + bi \mid a^2 + b^2 = 1\}$

$W : \mathbb{R} \to \mathbb{C}$, $W(x)$ = the pt. x radians from $(1,0)$ on unit circle.

$$\left.\begin{array}{l} \text{Fact. } W(x) = \cos x + i \sin x \\ W(xy) = W(x)W(y) \quad \text{(trig id)} \end{array}\right\} \quad \mathrm{Ker}(W) = \langle 2\pi \rangle$$

1st Isom Thm : $\mathbb{R} / \mathrm{Ker}(w) \cong w(\mathbb{R})$

$\sim \quad \mathbb{R} / \langle 2\pi \rangle \cong H$ circle group