MA312 Exam 2 Fall 2020 Answer 10 or more.

Start Time: ______________________________

End Time: ______________________________

Name/Honesty Signature: I did not use my notes, my text, or any outside resources ________________________

1. Let $f : G \to K$ be a homomorphism. Prove that $ker(f)$ is a normal subgroup of G.

2. Answer these questions on symmetric groups.

   (a) How many elements of order 3 are there in S_4?

   (b) Compute this product in S_4. (314)(124)(13)

3. Show that $f(x) = \log_a(x)$ is an isomorphism from $(\mathbb{R}^+, *)$ to $(\mathbb{R}, +)$.

4. Show that no group can have exactly two elements of order two.

5. Show that $SL(2,\mathbb{R}) \triangleleft GL(2,\mathbb{R})$ and the factor group $GL(2,\mathbb{R})/SL(2,\mathbb{R})$ is isomorphic to some very familiar group. What is this group? (Justify your answer.)

6. Prove that $D_4/Z(D_4)$ is isomorphic to $Z_2 \bigoplus Z_2$.

7. Let $G = U(10) \bigoplus Z_{15} \bigoplus S_4$. Find the order and the inverse of (2,4,(143)(12)).

8. Describe the group of rotational symmetries of the cube in relation to S_4. In particular, associate different symmetries of the cube to elements of order 1, 2, 3 and 4 in S_4.

9. Prove that if two cycles are conjugate in S_4 then they have the same cycle length.

10. If $H < G$ and $|G : H| = 2$ show that $H \triangleleft G$. Use this to prove that $A_4 \triangleleft S_4$.

11. Show that A_4 (the Alternating Group of 4 elements) does not contain an element of order six.

12. My favorite part of abstract algebra is ______________________________.