Groups = Symmetry.

We use the term "symmetry" to represent:
the way in which an object can be transformed yet remain invariant

Today we study the symmetries of a square.
Labels are only to distinguish symmetries,

Q: How many "symmetries" does a square possess 8.

We call this group D_4. It has $2 \cdot 4 = 8$ elements

$$D_4 = \{ R_0, R_{90}, R_{180}, R_{270}, D, D', H, V \}$$

off-diagonal flip

vertical flip

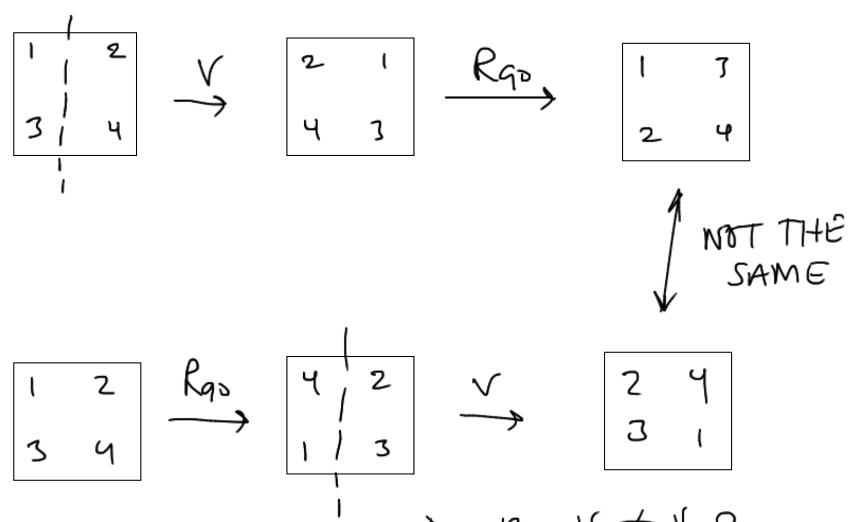
horizontal flip

counter-clockwise rotation by $0^\circ, 90^\circ, 180^\circ, 270^\circ$
(identity

main diagonal flip

Q: Is this group Abelian? (Do every two elements commute)
Niels Abel.

NO



$R_{90} \circ V$

V came first, then R_{90}
(function composition
$f \circ g(x) =$

$\Rightarrow R_{90} \cdot V \neq V \cdot R_{90}$

$V \circ R_{90}$

Q: Are there any elts that commute?

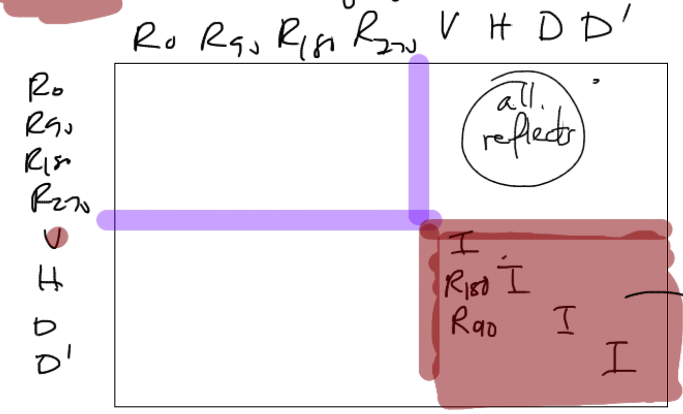
A: Yes, any two rotations.

Q: What are inverses of each elt?

(Recall, every elt. needs an inverse.

elt	inverse
R_0	R_0
R_{90}	R_{270}
R_{180}	R_{180}
R_{270}	R_{90}
V	V
H	H
D	D
D'	D'

Ex. Stare at Cayley Table for D_4 in Text.



all rotations

$H \cdot V = R_{180}$

$D \cdot V = R_{90}$

Garret: If $x \cdot y = e$ is $xy = yx$?

Ans: yes: If $x \cdot y = e$

$$x^{-1} \cdot x \cdot y = x^{-1} \cdot e$$

$$e \cdot y = x^{-1} \cdot e$$

$$y = x^{-1}$$

so mult. by x on right.

$$yx = \underbrace{x^{-1} x}_{e}$$

$$yx = e$$

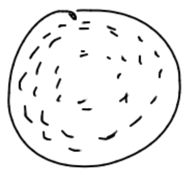
Example of an important infinite group

$GL(2,\mathbb{R})$ = generalized linear group:

= { 2×2 matrices w/ $\det \neq 0$, matrix mult. }

This set is <u>closed</u> under matrix mult

↳ mult. by two elements <u>in</u> the set produce a third elt. <u>in</u> the set.



<u>Ex.</u> $(\mathbb{Z}, \div)$ not closed b/c $2 \div 3 \notin \mathbb{Z}$.

Why is $\boxed{GL(2,\mathbb{R})}$ closed under $*$?

$$\det(A \cdot B) = \underbrace{\det(A)}_{\neq 0} \cdot \underbrace{\det(B)}_{\neq 0}$$

so $\neq$

0

$GL_n\mathbb{R}$:

$S = \{M \in M(n) \mid \det M \neq 0\}$
op. matrix mult

1. $\det A \cdot B = \det A \cdot \det B$

$$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} = \begin{pmatrix} a_1b_1 + a_2b_3 & a_1b_2 + a_2b_4 \\ a_3b_1 + a_4b_3 & a_3b_2 + a_4b_4 \end{pmatrix}$$

$*$

$$\det(A \cdot B) = \det A \cdot \det B$$

$\det A = a_1a_4 - a_2a_3$

$\det B = b_1b_4 - b_2b_3$

$\det A \cdot \det B = a_1a_4b_1b_4 - a_1a_4b_2b_3 - a_2a_3b_1b_4 + a_2a_3b_2b_3$

$\det * = (a_1b_1 + a_2b_3)(a_3b_2 + a_4b_4) - (a_3b_1 + a_4b_3)(a_1b_2 + a_2b_4)$

$= a_1b_1a_3b_2 + a_1b_1a_4b_4 + a_2b_3a_3b_2 + a_2b_3a_4b_4$

$- a_3b_1a_1b_2 - a_3b_1a_2b_4 - a_4b_3a_1b_2 - a_4b_3a_2b_4$

QED.

$(\det A^{-1}) = (\det A)^{-1}$

$GL_n\mathbb{R}$ is
CLOSED under matrix mult!