Friday – Week 6

#2 Ch. 7

$\langle \frac{3}{4} \rangle = \underbrace{\text{subgroup generated by } \frac{3}{4}}$

$\mathbb{Q}$ = set of fractions $\frac{p}{q}$, group under +

$= \{ 0, \pm\frac{1}{2}, \ldots, \frac{101}{100}, \frac{2}{1}, \ldots$

In $\mathbb{Q}$, $\langle \frac{3}{4} \rangle = \{ \frac{3}{4}, \frac{6}{4}, 0, \frac{-3}{4}, \ldots \frac{3n}{4} \ldots$ $\quad n \in \mathbb{Z}$.

$\frac{6}{4} = \frac{3}{2}$

$\mathbb{Q}^*$ = all the fractions, throw out zero, group under *

$= \{ 1, \frac{3}{4},$

$\langle \frac{3}{4} \rangle = \langle \frac{3}{4}, \frac{9}{16}, \frac{4}{3}, \frac{-27}{64}, \ldots \qquad \left(\frac{3}{4}\right)^n = \left\{ \frac{3^n}{4^n}, \frac{-3^n}{4^n} \right\}$

$= \langle \frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \ldots, \frac{4}{3}, \frac{16}{9}, \frac{64}{27}, \ldots, \frac{3^2 \cdot 4^1}{4^2 \cdot 3^1}, \cancel{\frac{3^n}{4^n} \cdot \frac{4^m}{3^m}}, \frac{3^{n-m}}{4^{n-m}}$

$\left(\frac{3}{4}\right)^{n-m}$

Tips for H/W

- If and only if: To prove iff statements do:

$A \Longleftrightarrow B$

prove:

$\Rightarrow$ Assume A prove B

⋮

$\Leftarrow$ Assume B prove A.

A is true if B is true

$\Updownarrow$

A is true ONLY if B is true

equiv. to

if B is not true A is not true.

if $\sim B$ then $\sim A$ iff A then B

contrapositive of statement

$C \Rightarrow D$

is

$\sim D \Rightarrow \sim C$

(contrapositive is equivalent to statement (original))

ORDER of an element

what does it mean for the order of elt. a to be n? $a^n = e$ $\Updownarrow$ n is the smallest power that kills a

Ex: $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$

$1^2 = 1+1$

$2^2 = 2+2$

Fact: $3^{666} = 0$ yet $|3| \neq 666$

Hint: (1) show $a^n = e$ (implies $|a| \leq n$)

(2) then assume $a^i = e$ w/ $i < n$, derive contradiction or show $i = 0$

Last Time: The identity element S_n is even

In S_n, $e = (12)(12)$

Thm: If $\pi \in S_n$ then π can't be both even & odd.

proof: If π is both even/odd:

π = even product of perms

π = odd product of perms

$\Rightarrow$ π^{-1} = even

π^{-1} = odd

Choose π = even, π^{-1} = odd

$\underbrace{\pi \cdot \pi^{-1}}_{e = \text{even by above.}}$ = even · odd = odd

$\times$

suppose

$\pi = (24)(45)$

$\pi = (12)(23)(45)$

$\pi = (24)(45)$

$\pi(45) = (24)\underbrace{(45)(45)}_{e}$

$\pi(45)(24) = (24)(24)$

$\pi\underbrace{(45)(24)}_{\text{this elt is } \pi^{-1}} = e$

$\Rightarrow \pi^{-1}$ is even

So S_n is split into even / odd elts.

- Identity is even
- even times even
- inverse of an even is even

$\Rightarrow$ Even's form a subgroup of S_n called the alternating group

$A_n \leq S_n$

Fact:

A_4 is the group of (rotations) symmetries of