Notes on Assignment 3

- 1. #22. Check carefully the assumption
- 2. If and only if requires two directions
- 3. Assignment 2 some comments in side comment box (open in Kami)

G has the property that if xy = zx then y = z

Prove G is ablian.

the (errant) strategy of many:

Better: Start w/ the truth!

then, let
$$x=a$$
 $y=ba$
 $z=ab$

(2)
$$\times y = 2 \times$$
 $6a = ds$.
(3) assumption on $G \Rightarrow y = 2$

Assume
$$xy = 2x$$
,

by $y = bx$
 $y = ab$

then
$$abc = abc$$

assumption \Rightarrow
 $y = 2$
 $bc = eb$.

Prod $P(e) = P(e \cdot e) = P(e) \cdot P(e)$ e = id in G Note $P(e) \in G$ e = id in G So

Ex. $Z_4 = 50, 1, 2, 3$? $H \leq D_4$, H = 5Ro, Rao, R₁₈₀, R₂₇₀? P_3 P_4 P_4 P_4 P_4 P_6 P_6

 $\frac{\varphi(e)}{\varphi(e)} = \frac{\varphi(e)}{\varphi(e)} \cdot \frac{\varphi(e)}{\varphi(e)} \cdot \frac{\varphi(e)}{\varphi(e)} = \frac{\varphi(e)}{\varphi(e)$

 \widehat{a} , $\varphi(\alpha^n) = \varphi(\alpha)^n \quad \forall \quad n \in \mathbb{Z}$

Ex n=2 $\varphi(a^2) = \varphi(a,a) = \varphi(a) \cdot \varphi(a) = (\varphi(a))^2$ By induction, true for $n \in \mathbb{Z}^+$.

If neo, 1.0, $\varphi(a^{-1}) = \varphi(a)^{-1}$

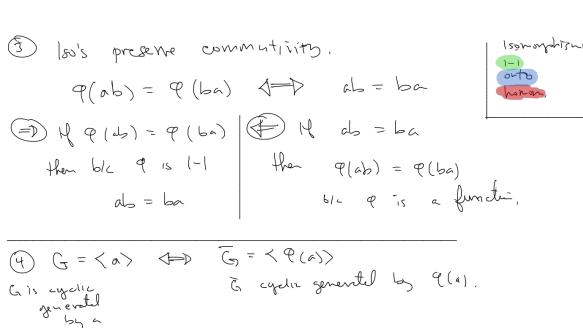
then -n>0 so, $\theta(\alpha^{-1})=\theta(\alpha)$

 $\varphi(e) = \varphi(a^n, a^{-n}) = \varphi(a^n), \varphi(a^{-n}) = \varphi(a^n), \varphi(a)^{-n}$

 $\overline{e} = P(\alpha^n) \cdot Q(\alpha)^n$ cancellation $Q(\alpha)^n = \frac{1}{e} Q(\alpha)^n$

 $\overline{e} \cdot \varphi(\alpha)^n = \varphi(\alpha^n) \cdot \overline{e}$ $\varphi(\alpha)^n = \varphi(\alpha^n)$

True & n



The G =
$$\langle \alpha \rangle$$
, then any element $g \in G$ looks like'

 $g = a^k$ for some $k \in \mathbb{Z}$.

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Does everything in G look like this?

Let $g \in G$ some random ett. in $g \in G$.

By property onto we know $g \in G$.

 $g \in G$

So $g \in G$
 g

Assume $G = \langle P(a) \rangle$. Show $G = \langle a \rangle$, let $a \in G$, by def. of Q, $Q(a) \in G$. Let $b \in G$ be anything. $Q(b) \subseteq G$ so

$$P(b) = P(a)^{k} = P(a^{k}) = \begin{cases} q \text{ is an isomorphism,} \\ by assumption \\ G = \langle q(a) \rangle, \end{cases}$$

$$because P \text{ is } 1-1$$

$$G = \langle a \rangle, \end{cases}$$

$$b = a^{k}.$$

Isomorphisms preserve orders.

thin $\varphi(\alpha^n) = \varphi(\alpha)^n = \overline{\varrho}$ $\frac{1}{2} n = \overline{\varrho}$ $\frac{1}{2} n = \overline{\varrho}$

If and only It the statement P if and only if Q, means
Pid Q, i-e, H a thon P AP only if a.

A If not a, then not P. (NADNP) I contrap ositive 0 nn € 9nn Ex. Show G abelian iff (ab) = o'b' (if and only of)
Assume G abelian, Show (ab) = a'b" Assume (ab) = a b show G is abelian