Wed, Week 7

- HW due Next Wed, 8 am
- HW #3 – graded (grades are in your folder (.pdf)
  #4 – this week

Ch.4
#20

Hint: know if $a, b \in G$, $|a| = 1, 5, 7, 35$

$|b| = 1, 5, 7, 35$

(1)

(2) G is Abelian, you then know something about $|ab|$

i.e., $(ab)^n = a^n b^n$

#46 (Ch 5) Hint:

$$\underset{\{1\}}{A_2} \leq A_3 \leq A_4 \leq A_5 \leq A_6 \leq \dots$$

$A_2 = \{(1)\}$

$\{1,2\}$

$S_2 =$ symm group on $\{1,2\}$

$S_2 = \{(1), (12)\}$

(12) not even

(1) A_2

look for two non-commutting eltr in A_4. they won't commute in A_n, $n>4$.

Isomorphisms

Last time: Isomorphism = bijective (1-1, onto), homomorphism $f(ab) = f(a)\cdot f(b)$

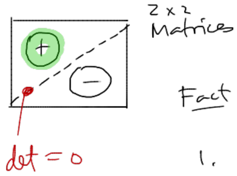
Important Example: $SL_2\mathbb{R}$ — groups of 2×2 real-valued matrices w/ det $=1$

S: det $=1$
L: linear (matrix)
$\mathbb{R}$: real entries
2: 2×2

Conjugation in $SL_2\mathbb{R}$ is an isomorphism

(Fix some $M \in SL_2\mathbb{R}$ to conjugate by M is to do this: for each other matrix A, form MAM^{-1}.

Fact: $\det(MAM^{-1}) = \underbrace{\det(M)}_{\text{in } SL_2\mathbb{R}} \cdot \underbrace{\det(A)}_{=1\ (A\in SL_2\mathbb{R})} \cdot \underbrace{\det(M^{-1})}_{\text{in } SL_2\mathbb{R}} = 1\cdot 1\cdot 1 = 1$



Fact: Conjugation in $SL_2\mathbb{R}$ is an isomorphism

1. Homomorphism:
let $\varphi_M : SL_2\mathbb{R} \longrightarrow SL_2\mathbb{R}$ by $\varphi_M(A) = MAM^{-1}$

$\varphi_M(AB) = MABM^{-1}$

$= \underbrace{MAM^{-1}}\underbrace{MBM^{-1}}$ (Insert I, identity matrix $I = M^{-1}M$

$= \varphi_M(A)\cdot\varphi_M(B)$ ✓

2. 1-1 If $\varphi_M(A) = \varphi_M(B)$ show $A=B$

$\varphi_M(A) = MAM^{-1} = MBM^{-1}$

$M^{-1}MAM^{-1} = M^{-1}MBM^{-1}$

$AM^{-1} = BM^{-1} \Rightarrow A = B$

$\uparrow \quad\quad \uparrow$
$M \quad\quad M$

3. onto Let $B \in$ Target Space, $B \in SL_2\mathbb{R}$.
Find some $A \in SL_2\mathbb{R}$ s.t

$\underbrace{\varphi_M(A)}_{\text{def}} = B$

$MAM^{-1} = B$ (Find some matrix A s.t. this is true)

$\downarrow$

$A = M^{-1}BM$ (also lives in $SL_2\mathbb{R}$)

in order for $MAM^{-1} = B$
we must have
$M^{-1}(MAM^{-1}) = M^{-1}(B)$
$AM^{-1} = M^{-1}B$
similarly $(AM^{-1})M = (M^{-1}B)M$
$A = M^{-1}BM$

Let $B \in SL_2\mathbb{R}$. Solve eqn $\varphi_M(A) = B$

$\Leftrightarrow MAM^{-1} = B$

Find some $A \in SL_2\mathbb{R}$ that works

· If $A = M^{-1}BM$ then $M(M^{-1}BM)M^{-1} = B$

Cayley's Theorem: Every finite group is just a group of permutations

S_3 elements: $(12)(13)$

D_4

	R_0	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_0	R_0	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_{90}	R_{90}	R_{180}	R_{270}	R_0	D'	D	H	V
R_{180}	R_{180}	R_{270}	R_0	R_{90}	V	H	D'	D
R_{270}	R_{270}	R_0	R_{90}	R_{180}	D	D'	V	H
H	H	D	V	D'	R_0	R_{180}	R_{90}	R_{270}
V	V	D'	H	D	R_{180}	R_0	R_{270}	R_{90}
D	D	V	D'	H	R_{270}	R_{90}	R_0	R_{180}
D'	D'	H	D	V	R_{90}	R_{270}	R_{180}	R_0

① Every elt of D_4 gives a permutation of the elements of D_4 (by mult. (on left))

$R_{90} \cdot R_0 =$

$R_{90} \longleftrightarrow T_{R_{90}} = (R_0\, R_{90}\, R_{180}\, R_{270})(H\, D'\, V\, D)$ viewing R_{90} as a permutation of elements of D_4

a permutation

$H \longleftrightarrow T_H = (R_0 H)(R_{90} D)(R_{180} V)(R_{270} D')$

$R_{90} \cdot H = D' \longleftrightarrow T_{D'} = (R_0 D')(R_{90} H)(R_{180} D)(R_{270} V)$

$R_{90} \longleftrightarrow (R_0\, R_{90}\, R_{180}\, R_{270})(H\, D'\, V\, D)$

$H \longleftrightarrow (R_0 H)(R_{90} D)(R_{180} V)(R_{270} D')$

$\updownarrow$

$R_{D'}$ $\quad (R_0 D')(R_{90} H)(R_{180} D)(R_{270} V)$ ✓

$(R_0\, R_{90}\, R_{180}\, R_{270})(H\, D'\, V\, D)(R_0 H)(R_{90} D)(R_{180} V)(R_{270} D')$

$= (R_0 D')(R_{90} H)$