Monday - Week 8

- Wed. 8 am (HW)
- Wed. Take Home midterm

HW Questions

$$\begin{tikzcd}
& \mathbb{Z}_6 \arrow[dl, dash] \arrow[dr, dash] & \\
\langle 0,3 \rangle \arrow[dr, dash] & & \langle 0,2,4 \rangle \arrow[dl, dash] \\
& \langle 0 \rangle &
\end{tikzcd}$$

$\leftarrow$ trivial subgroup

All subgroups of $\mathbb{Z}_6$

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① A_8: (non-cyclic subgroup of order 4): $V_4 = \{1, a, b, ab \mid a^2=1, b^2=1, ab^2=1\}$

✓ $(ab)^2 = abab = 1$

if $abab = 1$

$aba = 1$

$ab = a^{-1}$

but $a = a^{-1}$

② product of disjoint cycles

$a = (1235)(413) = (15)(234)$

Idea: write as disjoint cycles

Look in A_8 for a subgroup isomorphic to V_4:

Need:

- one elt. of order 2
- another elt. of order 2.
- their product

Reminder: If $x \in S_m$ then $|x| = n \Rightarrow x^n = 1$

(if x is a single cycle, you can write it: $x = (1, 2, 3, \ldots, n)$)

Order 2 elts: $(12), (13), (24) \ldots$

Also $(12)(34)$ has order 2.

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$a = (a_1 a_2 a_3 \dots a_n)$, what cycle is $(a_1 a_2 \dots a_n)^{-1}$?

123	(1)

Ex. In S_3.

a	a^{-1}
(12)	(12)
(123)	(132)

b/c $(12)(12)$

$(123)(\quad\quad)$ look @ mult. table

$S_3 = \{(1), (12)(13)(23)(123)(132)\}$

guess: Inverse is

$$\underbrace{(a_1 a_2 a_3 \dots a_n)}_{a}\underbrace{(a_1 a_n a_{n-1} a_{n-2} a_{n-3} \dots a_2)}_{\text{candidate for } a^{-1}} = (a_1)(a_2)$$

64, ch. 4

$U(2^n)$, $n \geq 3$ not cyclic

Ex $U(8)$
$U(16)$
$U(32)$

This element $2^n - 1$ lives in $U(2^n)$.

(B/c for all U-groups $U(k)$, the # $k-1$ is rel prime to k & less than)

Hint: $|2^n - 1|$ has order 2.

Note: Any cyclic group has at most one elt of order 2.

Find another elt of order 2 (hint: divisors of 2^n)

$(354) = (34)(35)$

even

In disjoint cycle

$(1243)(3521) = (13)(14)(12)(31)(32)(35)$

even

$1^{\underline{st}}$ $(1234) = (14)(13)(12)$

$(1243)(3521)$ 1 2

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 3 & 5 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 4 & 2 \end{bmatrix}$$

Thm: **Isomorphisms on Groups**

if $\varphi : G \longrightarrow \overline{G}$ is an isomorphism

1. φ^{-1} exists (1-1), φ^{-1} is an isomorphism too

2. G abelian / cyclic $\iff$ $\overline{G}$ abelian / cyclic

3. If $H \leq G$, $\varphi(H) \leq \overline{G}$

likewise

$$\overline{H} \leq \overline{G} \Rightarrow \varphi^{-1}(\overline{H}) \leq G$$