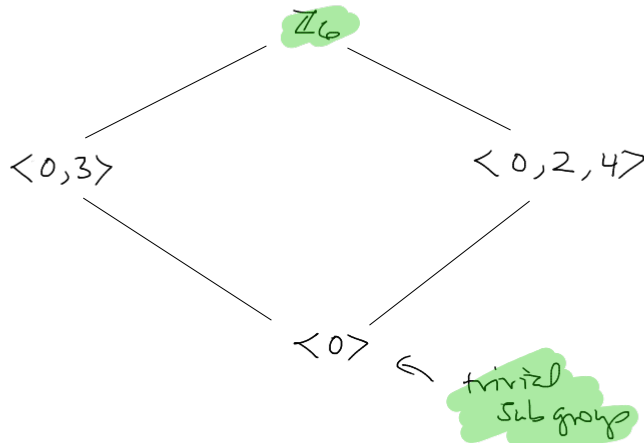


Monday - Week 8

- Wed. 8 an (HW)

- Wed. Take Home midterm

HW Questions



All subgroups
of \mathbb{Z}_6

$ab^2=1$

① A_8 : (non-cyclic subgroup of order 4) : $\mathbb{V}_4 = \{1, a, b, ab \mid a^2=1, b^2=1, (ab)^2=abab=1\}$

② product of disjoint cycles

$$a = (1235)(413) = (15)(234)$$

idea: write as disjoint cycles

Look in A_8
for a subgroup
isomorphic to

$$d_4 = L$$

$$ab = 1$$

\mathbb{V}_4 :

$$ab = a^{-1}$$

needs:

- one elt. of order 2

$$\text{but } a = a^{-1}$$

- another elt. of order 2.

- their product

Reminder: If $x \in S_n$

$$\text{then } |x|=n \Rightarrow x^n = 1$$

(if x is a single cycle, you
can write it:

$$x = (1, 2, 3, \dots, n)$$

Order 2 elts: $(12), (13), (24), \dots$

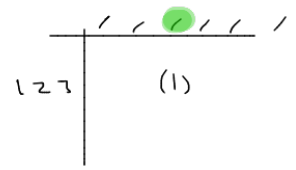
Also $(12)(34)$ has order 2.

22/
 $a = (a_1 a_2 a_3 \dots a_n)$, what cycle is $(a_1 a_2 \dots a_n)^{-1}$?

Ex. in S_3

a	a^{-1}
(12)	(12)
(123)	(132)

b/c (12)(12)
 (123)()



look @
 mult. table

$$S_3 = \{(1), (12), (13), (23), (123), (132)\}$$

guess: inverse is

$$\underbrace{(a_1 a_2 a_3 \dots a_n)}_a \underbrace{(a_1 a_n a_{n-1} a_{n-2} \dots a_2)}_{\text{candidate for } a^{-1}} = (a_1)(a_2)$$

64, ch. 4

$U(2^n)$, $n \geq 3$ not cyclic

Ex $U(8)$
 $U(16)$
 $U(32)$ | This $2^n - 1$ lives in $U(2^n)$,
element
(B/c for all U -groups $U(k)$, the # $k-1$
is rel prime to k $\frac{1}{2}$ less than)

Hint: $|2^n - 1|$ has order 2.

Note: Any cyclic group has at most one elt
of order 2.

Find another elt of order 2 (hint: divisors of 2^n)

$$(354) = (34)(35)$$

even

↑

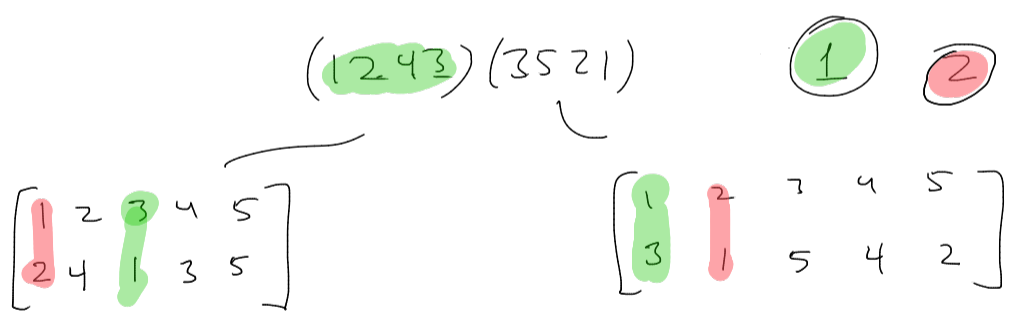
In disjoint cycle

$$(1243)(3521) = (13)(14)(12)(31)(32)(35)$$

even

1st

$$(1234) = (14)(13)(12)$$



Thm 1 Isomorphisms on Groups

if $\varphi: G \rightarrow \overline{G}$ is an isomorphism

① φ^{-1} exists (1-1), φ^{-1} is an isomorphism too

② G abelian $\iff \overline{G}$ abelian
cyclic \iff cyclic

③ If $H \leq G$, $\varphi(H) \leq \overline{G}$

likewise

$$\overline{H} \leq \overline{G} \implies \varphi^{-1}(\overline{H}) \leq G$$