Chapter 7 - Cosets & Lagrange's Theorem.

invented by Évariste Galois!

Cosets: Fix a subgroup H in G, then choose some $a \in G$,

$H =$

$aH = \{ a \cdot h \mid h \in H \}$

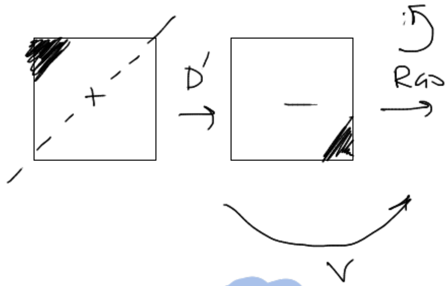
"left coset of H represented by a"

right coset : $Ha = \{ ha \mid h \in H \}$

Ex. Let $G = D_4$. Fix $H = \langle D' \rangle = \{D', e\}$ | $D_4 = \{R_0, R_{90}, R_{180}, R_{270}, H, V, D, D'\}$

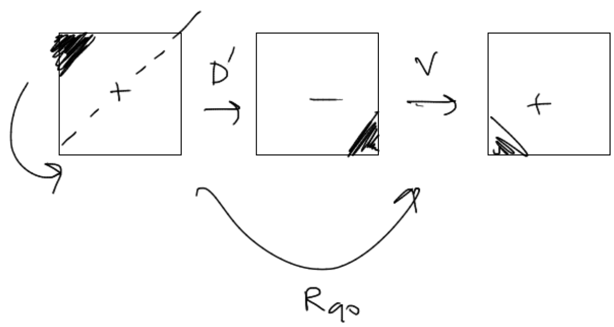
Let $a = R_{90}$.

$aH = R_{90} \cdot H = \{R_{90} \cdot D', R_{90} \cdot e\} = \{V, R_{90}\}$



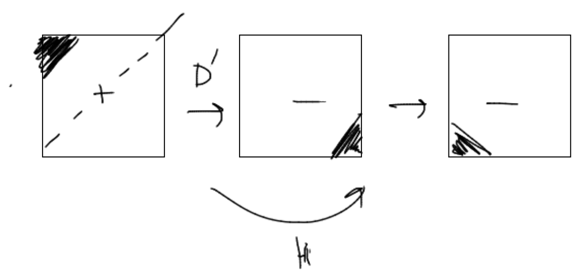
$V \cdot H = \{V \cdot D', V \cdot e\} = \{R_{90}, V\}$

$R_{90} H = V \cdot H$



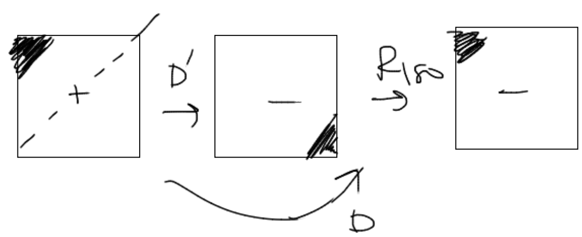
$R_{270} \cdot H = \{R_{270} \cdot D', R_{270} \cdot e\} =$

$= \{H, R_{270}\}$



$R_{180} \cdot H = \{R_{180} \cdot D', R_{180} \cdot e\} =$

$= \{D, R_{180}\}$



$D' \cdot H = \{D' \cdot D', D' \cdot e\} = \{e, D'\} = H$.

$$A_4 = \{(1),(123),(132),(124),(142),(134),(143),(234),(243),(12)(34),(13)(24),(23)(14)\}$$

note:

$$(123) = (13)(12)$$

$$H = \langle (123) \rangle = \{(123),(132),(1)\} = \{\underbrace{(132),(1)},(123)\}$$

$$(1)H = \{(1)(123),(1)(132),(1)(1)\} = H = (123)H = \{(123)(123),(123)(132),(123)(1)\}$$

$$\boxed{\begin{array}{l} 1.\ aH = H \iff a \in H \\ 2.\ aH = bH \iff a \in bH \quad \text{(Coset invariant of choice of representative)} \\ 3.\ |aH| = |bH| \end{array}}$$

$$(124)H = \{(124)(123),(124)(132),(124)(1)\}$$
$$= \{(14)(23),(134),(124)\}$$

$$(142)H = \{(142)(123),(142)(132),(142)(1)\}$$
$$= \{(234),(13)(24),(142)\}$$

$$(143)H = \{(143),(243),(12)(34)\} = \underbrace{(12)(34)H}_{\{(12)(34)(123),\,(12)(34)(132),\,(12)(34)\}} = (243)H$$

$$= \{(243),(143),(12)(34)\}$$