

Corrections and Clarifications for
Introduction to Topology: Pure and Applied

by Colin Adams and Robert Franzosa

We would like to thank all of the readers who have alerted us to needed corrections. Please contact us if you find other corrections, clarifications, or typographical errors that should be addressed. We can be reached by e-mail at: colin.c.adams@williams.edu, robert.franzosa@umit.maine.edu.

This list does not include minor inconsequential typographical errors.

Page 11, Example 0.3. The union and intersection should read $\bigcup_{t \in T} B_t$ and $\bigcap_{t \in T} B_t$.

Page 15, Definition 0.10 and the paragraph following it. There are two instances where $A \in \mathbb{R}^n$ should read $A \subset \mathbb{R}^n$.

Page 22, Example 0.17. The range of g and the range of $g \circ f$ should be \mathbb{R} rather than \mathbb{R}_+ .

Page 43, Exercise 1.34. This exercise is incorrect as stated because the discrete topology on the five-point set is the only Hausdorff topology on the set. A suitable replacement would be:

1.34. Prove that on a finite set, the discrete topology is the only topology that is Hausdorff.

Page 63. The following statement is incorrect: “For example, in the finite complement topology on \mathbb{R} , every sequence with an infinite range converges to every point in \mathbb{R} .” Replace it with:

For example, in the finite complement topology on \mathbb{R} , every sequence in which no element repeats more than finitely many times converges to every point in \mathbb{R} .

Page 64, Exercise 2.14(d). This exercise is incorrect. Replace it with:

- (d) Prove that every sequence in which no element repeats more than finitely many times converges to every point in \mathbb{Z}_+ in this topology.
- (e) Find a sequence with infinite range that converges only to 1 in this topology.

Page 64, Exercise 2.19. This exercise is incorrect. Replace it with:

- 2.19.** In these exercises assume that \mathbb{R} has the finite complement topology.
- (a) Prove that every sequence in which no element repeats more than finitely many times converges to every point in \mathbb{R} .
 - (b) Find a sequence with infinite range that converges only to 0.
 - (c) Find a sequence that has infinite range but does not have a limit.

Page 66, Proof of Theorem 2.14. “By Theorem 2.6 implies...” should read “Theorem 2.6(ii) implies...”.

Page 67, Example 2.19. It should be $\text{Int}(A) = [-1, 1)$ and $\partial(A) = \{1\}$.

Page 81, Exercises 3.4, 3.5, and 3.6. In each case determine whether the subset is open, closed, both, or neither in the corresponding topology.

Page 81, Exercises 3.11(a). This exercise is incorrect. Replace it with:

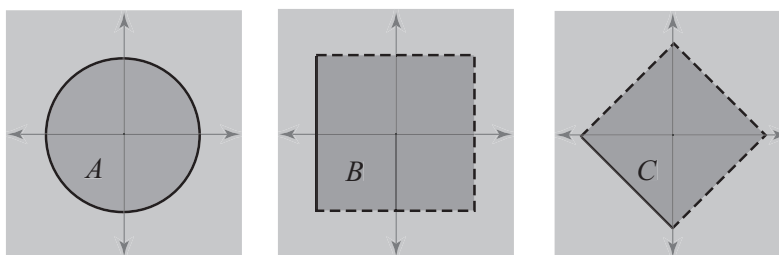
(a) Explore the relationship between $\text{Int}_A D$ and $A \cap \text{Int}_X D$. For each containment \subset and \supset , either prove that it holds or find a counterexample.

Page 87, Exercise 3.21. A modified, clarified version:

Consider the sets A , B , and C , illustrated in the figure below. A is the disk in the plane. B is the set $[-1, 1) \times (-1, 1)$, and

$$C = \{(x, y) \mid -1 \leq x + y < 1 \text{ and } -1 < x - y < 1\}.$$

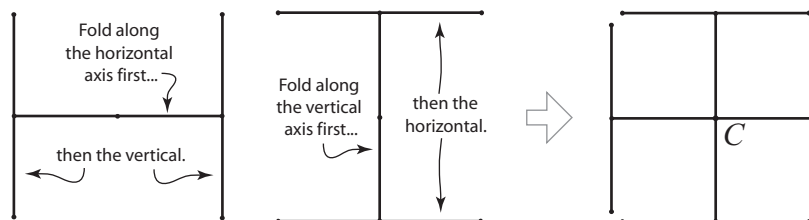
Determine whether or not each set is open, closed, both, or neither in each of the product topologies on the plane given by $\mathbb{R} \times \mathbb{R}$, $\mathbb{R}_l \times \mathbb{R}$, and $\mathbb{R}_l \times \mathbb{R}_l$, where \mathbb{R}_l is the real line in the lower limit topology.



Page 93, Example 3.19. In the definitions of B_y and B_y^* the inequalities should be $0 \leq y \leq 1$.

Page 107, Example 3.31. The configuration space is incorrect. Here is a correct description:

The space of configurations corresponding to folding along the horizontal axis first, then (if possible) folding along the vertical axis is shown on the left in the figure below. The space of configurations corresponding to folding along the vertical axis first, then along the horizontal axis is shown in the middle. In each of these spaces, the center point represents the unfolded configuration and therefore the center points coincide, resulting in the configuration space C shown on the right in the figure. It is important to realize that the “corner” points do not coincide. For example, a fold along the vertical axis to angle 0, followed by a fold along the horizontal axis to angle 0, does not result in the same final folded object as the same pair of folds in the opposite order (unless we then allow further rigid motion of the folded object).



Followup exercises: Assume that after the folding, we allow rigid motion of the folded sheet metal in 3-space.

(a) On a picture of the configuration space, indicate which configuration-space points are subsequently identified with each other because they result in the same final folded

object.

(b) Make a sketch of the quotient space obtained by making the identifications in (a).

(c) Now assume that we can distinguish the front and back of the piece of sheet metal (e.g., the front is painted white, and the back is painted black). On a picture of the configuration space, indicate which configuration-space points are subsequently identified with each other because they result in the same final folded and colored object.

(d) Make a sketch of the quotient space obtained by making the identifications in (c).

Page 122, Proof of Theorem 4.13. There are a couple of minor errors and there is some potential variable confusion. Here is an improved presentation of that proof:

Proof of Theorem 4.13. Let $U \subset \mathbb{R}$ be an open set. We prove that for each $x \in f^{-1}(U)$ there exists an open set $V_x \subset X$ such that $x \in V_x \subset f^{-1}(U)$. Let $x \in f^{-1}(U)$ be arbitrary, and pick $\varepsilon > 0$ such that $(f(x) - \varepsilon, f(x) + \varepsilon) \subset U$. By uniform convergence we can pick $N \in \mathbb{Z}_+$ such that $|f_n(s) - f(s)| < \frac{\varepsilon}{3}$ for every $n \geq N$ and $s \in X$. Pick $n' \geq N$. Let $U' = (f_{n'}(x) - \frac{\varepsilon}{3}, f_{n'}(x) + \frac{\varepsilon}{3})$, and let $V_x = f_{n'}^{-1}(U')$. We claim that V_x is open in X , contains x , and satisfies $f(V_x) \subset U$. (See Exercise SE 4.21.) Given the claim, it follows that for every $x \in f^{-1}(U)$, there exists an open set V_x in X such that $x \in V_x \subset f^{-1}(U)$. Therefore $f^{-1}(U)$ is open in X , implying that f is continuous. \square

Page 146. At the bottom of the page, the sentence beginning “Furthermore, justifying the notation....” is incorrect. Please delete it from your copy of the text.

Page 150, Exercise 5.14(b). This exercise is incorrect. Replace it with:

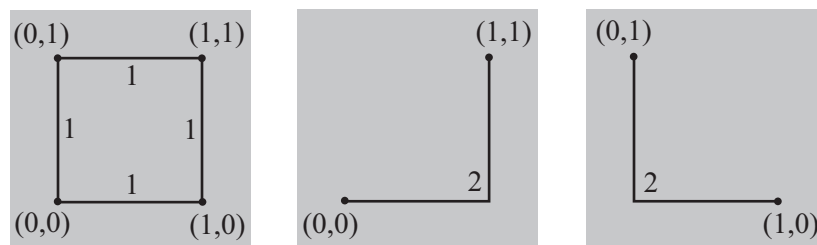
(b) In \mathbb{R}^n with the standard metric d , show that for $\varepsilon > 0$ and $x \in \mathbb{R}^n$, the closed ball $\bar{B}_d(x, \varepsilon)$ is the closure of the open ball $B_d(x, \varepsilon)$.

(c) Provide an example of a metric space in which the closed balls are not necessarily the closure of the corresponding open balls.

Page 152, Proof of Theorem 5.10. “ $= d - 1$.” should be “ $\leq d - 1$.”

Page 155, Exercise 5.19. In the definition of **corrects n errors** change “a unique” to “at most one”.

Page 160, Figure 5.9. The figure should appear as:



Page 176, Exercise 6.7(b). Where it refers to Example 6.9, it should be referring to Example 6.10.

Page 178, Lemma 6.16 and SE 6.14-6. The assumption that X is connected is unnecessary.

Page 184, Definition 6.21. The assumption that X is connected is unnecessary.

Page 192, the comment following Definition 6.27. Insert “in the standard metric” at the end of the sentence.

Page 192, Example 6.23. In both instances $(0, \infty)$ should be $[0, \infty)$. We can view A as the image of the embedding $f : [0, \infty) \rightarrow \mathbb{R}^2$ given (in (r, θ) polar coordinates) by $f(\theta) = (\frac{\theta}{\theta+1}, \theta)$, and therefore, as asserted in the second paragraph, A is the image of the connected space $[0, \infty)$ under a continuous function.

Page 195, Proof of Theorem 6.29. $f \circ g$ is really a path in Y , but since the image of $f \circ g$ is a subset of $f(X)$, by restricting the range of $f \circ g$, the result is a path in $f(X)$.

Page 197, Exercise 6.44. There is a better approach to the proof than that suggested by the provided hint.

Alternate hint: Given $x, y \in \mathbb{R}^n - C$, find a path in $\mathbb{R}^n - C$ going from x to y in a plane in \mathbb{R}^n containing x and y .

Page 203, Exercise 6.56. Replace “that they cannot occupy” with “that neither robot can occupy”.

Page 209, Proof of Theorem 7.6, & Page 212, Exercise 7.2. Although Theorem 7.6 is presented prior to Theorems 7.7 and 7.8, the intent is to have Theorems 7.7 and 7.8 available in the exercise to prove Theorem 7.6.

Page 212, Exercise 7.7. Replace with:

7.7. The Dirichlet Prime Number Theorem indicates that if a and b are relatively prime, then the arithmetic progression $A_{a,b} = \{\dots, a-2b, a-b, a, a+b, a+2b, \dots\}$ contains infinitely many prime numbers. Use this result to prove that \mathbb{Z} in the arithmetic progression topology is not compact.

Page 213, Hint to Exercise 7.13(a). $X - G$ should be $(X \times Y) - G$.

Page 215, in (ii) near the top of the page. The N in the exponent in the denominator should be n .

Page 224, Proof of Lemma 7.27, first sentence. “any real number λ ” should be “every $\lambda > 0$ ”.

Page 227, Proof, second paragraph. After $\frac{4}{9}$ insert “for all $x \in A$,” but this paragraph would be more clear with the following statements in place of the current versions:

$$|f(a) - g_1(a)| \leq \frac{2}{3} \text{ for all } a \in A,$$

$$|f(a) - g_1(a) - g_2(a)| \leq \frac{4}{9} \text{ for all } a \in A.$$

Page 227, in the portion of the proof between SE 7.30 and SE 7.31. Where it refers to Theorem 7.20 it should be referring to Theorem 7.18.

Page 229, Example 7.15. It is not necessary to assume that s is an element of A with minimum absolute value. The argument carries through if s is simply any element in the set A .

Page 232, item (iv). “Continuous functions” should say “Continuous real-valued functions”.

Page 235, Proof of Theorem 7.41. The collection $\{O, V_1, \dots, V_n\}$ should be $\{U, V_1, \dots, V_n\}$.

Page 243, Example 8.5. The topological conjugacy should be:

$$h(x) = \begin{cases} -|x|^{\log_2(3)} & \text{if } x < 0, \\ x^{\log_2(3)} & \text{if } x \geq 0. \end{cases}$$

Pages 245-6, Exercise 8.9. In each subexercise, each $f(x)$ should be $h(x)$.

Page 247, Definition 8.7 and the comment following it. The definition and comment pertain to the situation where f is continuous, but the assumption that f is continuous is not made here. With no assumption about the continuity of f , the definition and comment should read:

DEFINITION 8.7. Given $f : X \rightarrow X$, assume that x^* is a period- m point of f . We say that x^* is a **stable periodic point**, or **has a stable periodic orbit** if $x^*, f(x^*), \dots, f^{m-1}(x^*)$ are all stable as fixed points of f^m . We similarly define **asymptotically stable** for periodic points and periodic orbits. If x^* is a stable periodic point, but not asymptotically stable, we say that it is a **neutrally stable periodic point** or has a **neutrally stable periodic orbit**. If x^* is not a stable periodic point we say that it is an **unstable periodic point** or has an **unstable periodic orbit**.

In Definition 8.7, in the case where f is continuous, the stability of x^* as a periodic point is solely determined by its stability as a fixed point of f^m . Specifically, if f is continuous and x^* is a stable fixed point of f^m , then so are

$$f(x^*), f^2(x^*), \dots, f^{m-1}(x^*).$$

The same holds for x^* being an asymptotically stable periodic point, a neutrally stable periodic point, and an unstable periodic point. (See Exercise 8.17.)

Page 251, Example 8.9. “so 1/2 is a stable fixed point.” should read “so 1/2 is an asymptotically stable fixed point.”

Page 253, first sentence. “and x_0 is stable.” should read “and x_0 is asymptotically stable.”

Page 253, Exercise 8.17. The function f should be assumed to be continuous throughout.

Page 256, first paragraph. While we introduce sensitive dependence on initial conditions in this section, it is not until Section 8.5 that we present the theorem demonstrating that every continuous chaotic function on an infinite metric space has sensitive dependence on initial conditions.

Page 258, Proof of Lemma 8.12. “since $a_j - b_j \leq 1$ ” should be “since $|a_j - b_j| \leq 1$ ”.

Page 259, Theorem 8.14 and the paragraph preceeding it, & Page 261, Exercise 8.25. It should be assumed that the function f is continuous.

Page 262, Lemma 8.17. The second part of the expression for $T^n(x)$ should be $-2^n x + 2j$ rather than $-2^n x + 2^n + 2j - 2$.

Page 262, Proof of Theorem 8.18. The proof does not end until the completion of Exercise SE 8.35.

Page 263, last paragraph. “parameter values $[0, 4]$ ” should read “parameter values $\alpha \in [0, 4]$ ”.

Page 270, Proof of Theorem 8.19. In the fourth paragraph there are three instances where “greater than $\delta_0/2$ ” should be “at least $\delta_0/2$ ”. In the three-line expression of inequalities and equality on the next page, the second inequality should be $<$ rather than \leq . And in the paragraph following that, there are two corrections to the first sentence; it should read:

Since the distance between x and the orbit of q is at least 4δ , it follows that $4\delta \leq d(x, f^{hm-k}(q))$.

Page 272, Exercise 8.46. There are two corrections in this exercise. In (a), in the definition of C , the expression “ $< \delta$ ” should be “ $\leq \delta/2$ ”. In (c), the sensitivity constant δ^* should be $\delta^*/2$.

Page 274, Example 9.1. Where it says “the homotopy $F(x, t)$ translates” it should say “the function $F|_{X \times \{t\}}$ translates”.

Page 274, Example 9.2. In defining $F(x, t)$, the expression $1 - 2t$ should be $2t - 1$.

Page 279, Proof of Theorem 9.7. The last two sentences should end with “for some $n \in \mathbb{Z}$ ”.

Pages 286-7. The notions of a point of discontinuity and set of discontinuities are used, but they were not previously defined. One remedy is to introduce the following definition and theorem:

DEFINITION. Given a function $f : X \rightarrow Y$ and a point $x \in X$, then f is **continuous** at x if for every open set V containing $f(x)$ there exists an open set U containing x such that $f(U) \subset V$. If f is not continuous at $x \in X$, then x is a **point of discontinuity** of f , and the set of all points of discontinuity of f is the **set of discontinuities** of f .

THEOREM. A function $f : X \rightarrow Y$ is continuous if and only if its set of discontinuities is empty.

Page 290, Exercise 9.18(a). It should be assumed that $c > 0$.

Page 292, Proof of Theorem 9.20. For clarification, the “aforementioned open balls” are those in the finite collection of open balls of radius $\varepsilon/2$ that cover $f(S^1)$.

Page 300, Proof of Theorem 9.29. Delete “ f ,” from “As a consequence of f , the Lebesgue Number Lemma, we can”.

Page 301, Proof of Theorem 9.30. “Theorem 9.26 implies that” should be “Theorem 9.25 implies that”.

Page 307, Proof of Theorem 10.3. “take the ray in \mathbb{R} ” should be “take the ray in \mathbb{R}^2 ”.

Page 309, Lemma 10.4. The assumption on the degree of h should be that it is *not* equal to 1, not that it equals 1.

Page 310, Exercise SE 10.12. “Show that $f(x^*) \in D$ ” should be “Show that $k(x^*) = ax^*$ implies that $f(x^*) \notin D$.”

Page 311, third paragraph. The bundle b should be denoted as \mathbf{b}^i .

Page 317, first paragraph. Replace $\mathbf{p}^* \cdot \mathbf{p}^* = 1$ with $\mathbf{p}^* \cdot \mathbf{p}^* \neq 0$.

Page 318, Example 10.7(i). “real numbers greater than x ” should be “real numbers greater than or equal to x ”.

Page 327, Example 10.10 and Equation 10.11. In the expressions for the optimal mixed strategies $P(\mathbf{q}, \mathbf{r})$ the equalities $p_2 = 0$, $p_4 = 0$, and $p_i = 0$ should be inequalities $p_2 \geq 0$, $p_4 \geq 0$, and $p_i \geq 0$.

Page 328, Example 10.11. In the equation $E = 10[2a(b+c-1)+c+b-3cb]$, the 3 should be a 4.

Page 344, just prior to Figure 11.11 and in the figure’s caption. The L_1 and L_2 should be J_1 and J_2 , respectively.

Page 347, Exercise 11.14(a). To clarify the context for this exercise, it should read: Assuming that L does not intersect S'' between b' and b , a path $aLa'S'b'Lb$ does not intersect S'' .

Page 376, Example 12.7. The bracket polynomial for the first example should be $-A^2 - A^{-2}$ instead of $A^2 - A^{-2}$.

Page 399, Definition 13.12 and Theorem 13.13. The graphs should be assumed to be connected.

Page 414, Proof of Theorem 13.26, second-to-last line. The G' and G should be G_2 and G_1 , respectively.

Page 421, Exercise 13.26(b). The G' and G should be G_2 and G_1 , respectively.

Page 421, Exercise 13.31. In two places, replace $n \geq 3$ with $n \geq 1$.

Page 429, Proof of Theorem 14.11. In construction step (iii) where it refers to step (b), it should be referring to step (ii).

Page 435, Exercise 14.1. In the parenthetical comment, “extra-point satisfies” should be “extra-point line satisfies”.

Page 439, just after Example 14.2. Where it refers to Theorem 14.18, it should be referring to Theorem 14.19.

Page 443, Example 14.4. Another approach to identifying a compact surface such as the one in this example is to use the relationships $K = P\#P$ and $T\#P = P\#P\#P$ to express the connected sum as a connected sum of projective planes only. In the case of $K\#K\#P\#P\#T\#T$, each K

contributes a $P\#P$ and each $P\#T$ contributes a $P\#P\#P$, so that the result is then $10P$.

Page 462. In four instances we misspelled Hantzsche as Handtche. Our apologies to Walter Hantzsche.

Page 462, Exercise 14.32. The identifications between each top face and each bottom face are incorrect. The top 4 and 5 faces should each be rotated 180 degrees (rather than reflected, as shown) when glued with the corresponding bottom faces. The following illustration correctly shows how the identifications are made:

