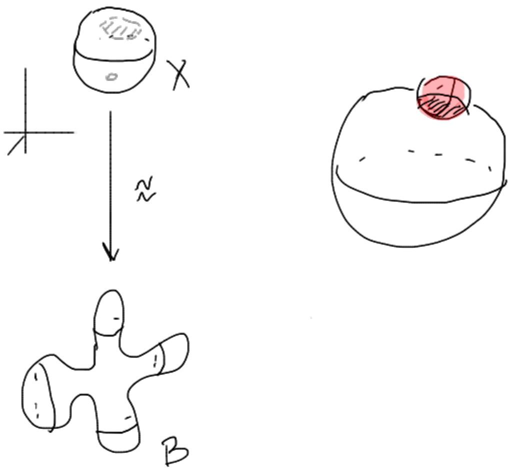
# Friday - Week 1

Topology ~ Rubber Geometry




$T_{std} = \{$ any open ball of radius $\epsilon$ in $\mathbb{R}^3$ intersected w/ $X \}$

## Topology on a set

Let $X$ = unit sphere
(set)

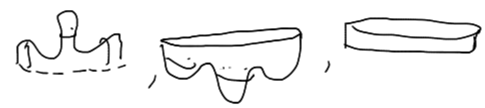
$T = \{ \phi, X,$  $\}$

annulus

$f: X \longrightarrow B$

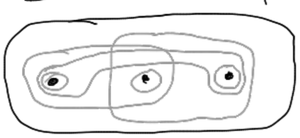
continuous & its inverse is cts

New topology on $B$

$T' = \{ \phi, B,$  $\}$

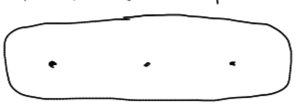
## Two extreme topologies

Ex: Discrete topology: $T_{Discrete} = \{$ every subset is open $\}$



(Fine!)

Ex Trivial Topology



$T_{trivial} = \{ \phi, X \}$

Finite Complement Topology on $\mathbb{R}$

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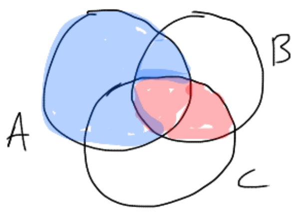
First: DeMorgan's Laws:




$$A-(B\cap C) = (A-B)\cup(A-C)$$

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For general cases:

$X =$ whole set

$A_i \subset X$

then

$$X - \bigcap_{i=1}^{n} A_i = X-(A_1\cap A_2 A_3 \dots \cap A_4)$$

$$= \bigcup_{i=1}^{n} (X-A_i)$$

$$X - \bigcup_{i=1}^{\infty} A_i = X-(A_1\cup A_2\cup A_3\cup\dots)$$

$$= \bigcap_{i=1}^{\infty} X-A_i$$

F.C.T. on $\mathbb{R}$

Set $X = \mathbb{R}$

$T = \{$ any subset whose complement is finite, or $\phi \}$

$= \{ A_i \subset \mathbb{R} \mid \mathbb{R} - A_i$ is finite $\}$

Ex: $\mathbb{R} \in T$ b/c $\mathbb{R} - \mathbb{R} = \{\phi\}$ a finite set

$\mathbb{R} - \{0\}$ is open b/c its complent is $\{0\}$

$A_1 = \mathbb{R} - \{1,2,3\}$ is open b/c $X - A_1 = \{1,2,3\}$.

$A_2 = \mathbb{R} - \{7,2,5\}$.

Verify: $A_1 \cap A_2 \in T$.

$A_1 \cap A_2 = \mathbb{R} - \{1,2,5,7,3\}$, still open

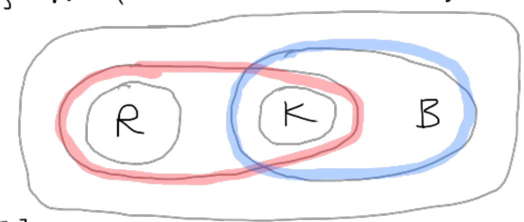
$A_1 \cup A_2 \overset{\text{DeMorgan's}}{=} \mathbb{R} - \{1,2,3\} \cup \mathbb{R} - \{7,2,5\} \overset{\text{D'M}}{=} \mathbb{R} - \{2\}$

Topology on a set.

Set $X$, $T = \{$ subsets of $X$ s.t.

- $\phi \in T$, $X \in T$
- finite intersections of subsets belonging to $T$ live in $T$
- any unions of subsets belonging to $T$ live in $T$.

Ex: let $X = \{$ Rachel, Kayla, Brenden $\}$




$T = \{\{R\}, \{K\}, \{R, K\}, \{K, B\}, \{R, K, B\}, \phi\}$