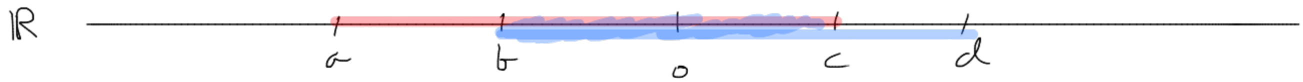
Topology, the definition evolved from studying $\mathbb{R}$.

The notion of a "topology on a set" ______________

indicates closeness.



an open interval, is denoted (a, b) has this property:

- $(a,c) \cap (b,d) = (b,c)$
- $(1,10) \cap (2,11) \cap (3,12) \cap (4,13) = (4,10)$

closure w/ finite intersections

- $S = \left\{ \left(1-\frac{1}{n}, 1+\frac{1}{n}\right) \middle| \, n \in \{2,3,\dots\} \right\}$

No closure w/ ∞-ints

examples of members of S:

$\left(1-\frac{1}{2}\ \ 1+\frac{1}{2}\right)$

$\left(1-\frac{1}{3}, 1+\frac{1}{3}\right)$

$\left(1-\frac{1}{4}, 1+\frac{1}{4}\right)$

Note: $\bigcap_{n=2}^{\infty} \left(1-\frac{1}{n}, 1+\frac{1}{n}\right) = \{1\}$ not open.

why not 1.000001?

- $\bigcup_{n=2}^{\infty} \left(1-\frac{1}{n}, 1+\frac{1}{n}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$ open
- $(1,2) \cup (4,5)$ union of two open sets.
- Here infinite unions of open intervals give open intervals or unions of them

Let X be a set. A collection of subsets T form a topology on X if:

1. Both ϕ & X are in T.
2. Finite intersections of elements of T, are in T
3. Any (finite or infinite) union of elements of T are in T.

Ex. $X = \mathbb{R}$

$T = \{$all open interals or unions thereof or ϕ or $\mathbb{R}\}$

def'n

(a,b) $a \in \mathbb{R}$ $b \in \mathbb{R}$

$(1,2) \in T$.

$\mathbb{R} = (-\infty, \infty)$

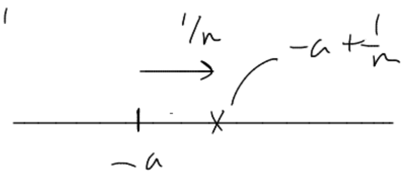
Note: $[a,b]$, sets of this form wouldn't work b/c

$\bigcup_{n=1}^{\infty} [\frac{1}{n}-a, \frac{1}{n}+a] = (-a, a+1]$

Form has changed.

why isn't $-a$ in $\bigcup_{n=1}^{\infty} [\frac{1}{n}-a, \frac{1}{n}+a]$

If $x \in \bigcup_{n=1}^{\infty} [\frac{1}{n}-a, \frac{1}{n}+a]$ then $x \in [\frac{1}{m}-a, \frac{1}{m}+a]$



if $-a \in [\frac{1}{m}-a, \frac{1}{m}+a)$ then $m = \infty$

Def'n: A subset A is open in a topological space (X,T) if $A \in T$. (If A is a subset member of T.)

Def'n: A set A is closed iff $X - A$ is open.

Topology, Week 1 Wed

Basic Concepts

1-1 : If $f(x) = f(y)$ then $x = y$

onto : $\forall y \in Y$, $\exists x \in X$ s.t $f(x) = y$

set : collection of elements, typically capital letters X

subset : $A \subset X$ means if $a \in A$ then $a \in X$

Three important types of sets

Finite	Countably Infinite	Uncountably Infinite
$\{1,2,3,\dots,n\}$	$\{1,2,3,\dots\}$	$\mathbb{R}, \mathbb{C}$
	$\mathbb{Z}$ = integers	$\mathbb{R}^2$
	$\mathbb{Q}$ = rational #'s	$\mathbb{R}-\{0\}$
	$\mathbb{N} = \{0,1,2,3,\dots\}$	$\mathbb{R} \sim \mathbb{Q}$ = Irrationals