Monday - Week 11

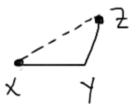
<u>Metric</u> : A metric on a space X is a distance function

$d : X \times X \longrightarrow \mathbb{R}$ → vary / depend on which metric is in play.

$d(x_1, x_2) =$ distance b/w x_1 & x_2

① Non-negative : $d(x,y) \geq 0$

② Symmetric : $d(x,y) = d(y,x)$

③ 

Triangle Inequality : $d(x,y) + d(y,z) \geq d(x,z)$

<u>Std Metric on $\mathbb{R}$</u> :

$X = \mathbb{R}$

$d(x,y) = |x-y|$

$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2 + (w_1-w_2)^2}$

Euclidean Metric in $\mathbb{R}^4$

$\mathbb{R}^1$: $\sqrt{(x_1-x_2)^2} = |x_1 - x_2|$

Verify this is a metric : (satisfy ① ② ③)

① $|x-y| \overset{\text{why}}{\geq} 0$

→ if $x > y \geq 0 \Rightarrow |x-y| = x-y \geq 0$

if $x < y$: $|x-y| = -(x-y) = y-x \geq 0$ if $y > 0$

There are other cases that work out similarly

Def'n $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

② $|x-y| = |y-x|$ why?

if $y < 0 < x$ then $x - y > 0$ so $|x-y| = x-y$

$y - x < 0$ so $|y-x| = -(y-x) = x-y$

other cases are similar

③ Δ-inequal : Show $|x-z| \overset{?}{\leq} |x-y| + |y-z|$

if $x \geq y \geq z$

$x - z \overset{?}{\leq} x-y + y-z = x-z$ yes

$|x-y| = x-y$

$|y-z| = y-z$

$|x-z| = x-z$

Taxi-Cab Metric or L^1-metric on $\mathbb{R}^2$

Let $\bar{x} = (x_1, x_2) \in \mathbb{R}^2$

$\bar{y} = (y_1, y_2) \in \mathbb{R}^2$

$$d_T(\bar{x}) = |x_1 - x_2| + |y_1 - y_2|$$

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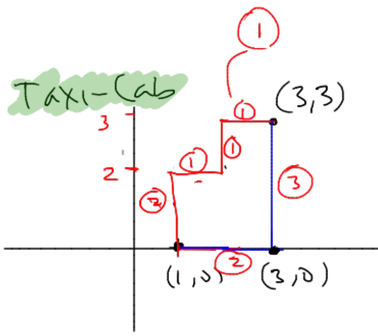
std metric: L^2-metric

$$= \left((x_1 - x_2)^2 + (y_1 - y_2)^2 \right)^{\frac{1}{2}}$$

$\leftarrow$ 2's replaced by 1's

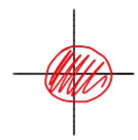
L^p-metrics are a thing.

Def'n: geodesic: path of shortest dist. b/w 2 points

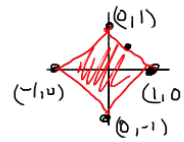


Note: unit-balls / unit spheres

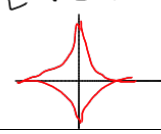
std-metric



L^1-metric



$L^{p \to 0}$-metric



$p << 0$