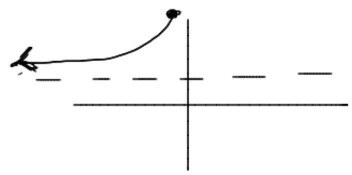
Homework Q's

$(-\infty, 0] \cong (a, b]$



think:

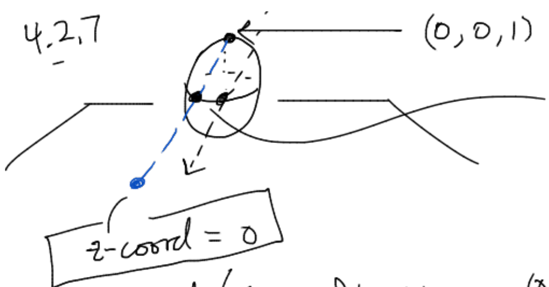
e^x

Find a homeo.

$0 \mapsto b$

$-\infty \mapsto a$

4.2.7



$(x, y, \underline{f(x,y)})$

$t\big((x,y,f(x,y)) - (0,0,1)\big) + (0,0,1) \qquad t \in \mathbb{R}$

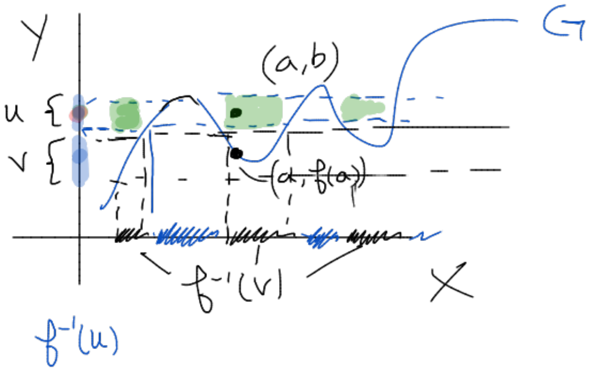
line thru north pole & $(x,y,f(x,y))$ on sphere.

sub

get a single point will correspond.

4.2.10

$f: X \xrightarrow{\text{cts}} Y$, $G = (x, f(x))$, Y = Hausdorff. Show G is closed.



pick $(a,b) \in X \times Y - G$ find nbhd about it disjoint from G.

$b \in Y$, $f(a) \in Y$ so Hausdorff $\Rightarrow$

$b \in \underset{\text{open}}{U}$, $f(a) \in \underset{\text{open}}{V}$ w/ $U \cap V = \phi$

V open, $f^{-1}(V)$ is open in X

$f^{-1}(V) \cap f^{-1}(U) = \phi$ so $f^{-1}(V) \times U$ is an open set containing (a,b).

$f^{-1}(V) \times U \cap G = \phi$. For ex, if (c,d) lived in the intersection the $f(c) \in V$, and $d \in U$ since U & V are disjoint $f(c) \neq d$

$\Rightarrow (c,d) \notin G$

4.9 $f, g : X \to Y$, Y Hausdorff $f(x) = g(x)$ $\forall$ $x \in D$.

Show $f = g$ $f(x) = g(x)$ $\forall$ $x \in X$

If two continuous functions, mapping into a Hausdorff space agree on a dense set, they are equal.

Ex: $f : \mathbb{R} \to \mathbb{R}$ & if $q \in \mathbb{Q}$, $f(q) = g(q)$ then $f = g$.

If $x \in X$, $x \in \overline{D}$ b/c D is dense.

If $f(x) \neq g(x)$ use Hausdorff $f(x) \in U$, $g(x) \in V$

$U \cap V = \phi$

Let $A = f^{-1}(U) \cap g^{-1}(V)$ is open. A is non-empty b/c of the density of D & assumption.

density $\Rightarrow \exists\, d \in f^{-1}(U) \cap D$, Assumption $\Rightarrow f(d) = g(d)$

yet $f(d) \in U$, $g(d) \in V$

$\otimes$.

Metric:

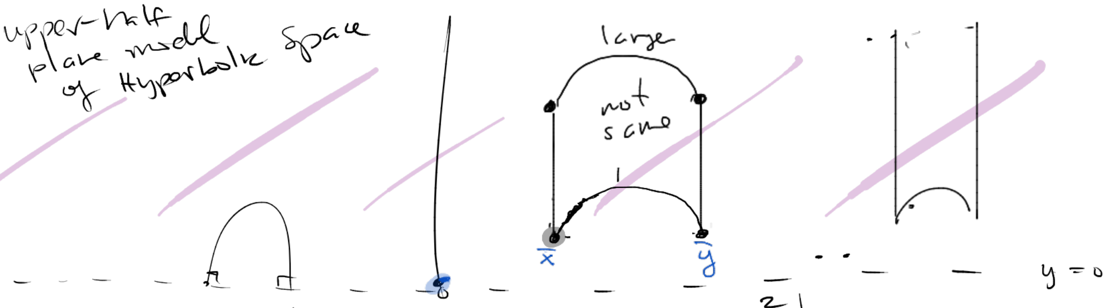
Every Metric Space is a topological space.

Metric Space: Set X w/ metric ρ. (X, ρ)

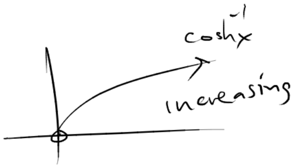
ϵ-balls: $B(x,\epsilon) = \{ y \in X \mid \rho(x,y) < \epsilon \}$ give a basis for a topology.

Cool Metric:

$\mathbb{H}$ = upper-half plane model of Hyperbolic space



$$d(\bar{x},\bar{y}) = \cosh^{-1}\left(1 + \frac{(x_1 - y_1)^2 + (x_2 - y_2)^2}{2x_2 \cdot y_2}\right)$$



$$= \ln\left(1 + \underline{\qquad\qquad}\right)$$