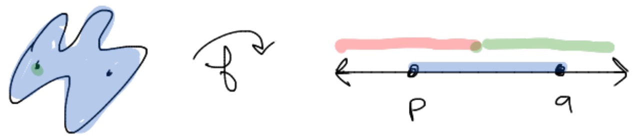
Friday - Week 12

IVT - general version:

For X, a connected top. space $f : X \longrightarrow \mathbb{R}$



if $p, q \in f(X)$ $\forall\, r \in \mathbb{R}$ $p \leq r \leq q$ you know

$$r \in f(X).$$

proof: If $r \notin f(X)$. Then $\overset{U}{\overset{\shortparallel}{(-\infty, r)}} \cup \overset{V}{\overset{\shortparallel}{(r, \infty)}}$ is open

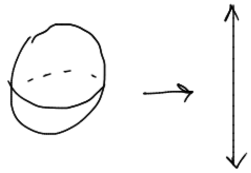
and it contains $f(X)$. $f(X) \cap U \neq \phi$ (b/c of p)

$f(X) \cap V \neq \phi$ (b/c of q)

$\Rightarrow f^{-1}((-\infty, r) \cup (r, \infty))$ is a separation of X,

which is impossible since X is connected

Thm: Let f be a cts function from S^2 to $\mathbb{R}$.



$\exists$ points $x, -x \in S^2$ such that ($-x$ is the antipodal point of x)

$$f(x) = f(-x)$$

proof: Let $g(x) = f(x) - f(-x)$. Claim $g(x)$ is continuous.

Case I $\exists$ $p \in S^2$ such $g(p) = 0$ then

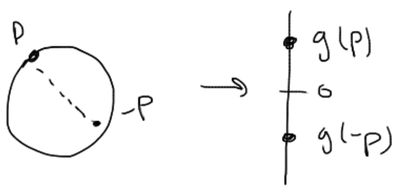
$$g(p) = f(p) - f(-p) = 0 \quad \text{so} \quad f(p) = f(-p)$$

(theorem Holds)

Case II Assume $g(p) \neq 0$.

Say $g(p) > 0$. Then

$$\underset{\displaystyle f(p) - f(-p)}{\overset{\parallel}{g(p)}} > 0 \quad \text{and note}$$



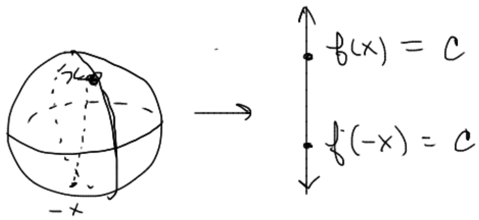
$$g(-p) = f(-p) - f(-(-p))$$
$$= f(-p) - f(p) = -(g(p))$$

$\Rightarrow$ $g(p)$ & $g(-p)$ have opposite signs.

So $\overset{\text{cts}}{g} : \underset{\text{connected}}{S^2} \to \mathbb{R}$ implies for $0 \in (\overset{-}{g(-p)}, \overset{+}{g(p)})$ it must be true that $\exists$ $q \in S^2$ s.t. $g(q) = 0$.

So $f(q) - f(-q) = 0$ or $\boxed{f(q) = f(-q)}$

Apply this theorem



Rephrase:
$f(x) =$ "Intensity of daylight @ location x

Assume the Earth is a sphere (topological)
Let $f(x)$ is the temperature @ $x \in$ Earth.

$-x$ = antipodal pt.

Interpret the previous theorem in this context

$f: S^2 \longrightarrow \mathbb{R}^2$

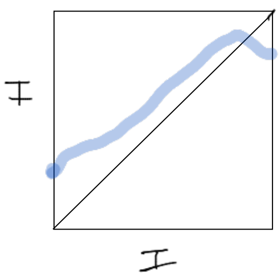
$f(x) =$ (temp, barametric pressure).

Brouwer's Fixed Point Theorem

<u>1-D</u>: Any cts map $f: \underset{= [-1,1]}{I} \longrightarrow \underset{= [-1,1]}{I}$ has a fixed point.

<u>Fixed Point</u>: some $y \in I$ s.t. $f(y) = y$

<u>Ex</u>:



f cts $\Rightarrow$ can't pick up pen w/ graph

gen'l proof:

let $g(x) = x - f(x)$

if $g(p) = 0$ done b/c $x = f(x)$

else $g(p) > 0$ (or < 0)

proceed as before.

① This is true for n-balls

$[-1,1]$ = unit ball in $\mathbb{R}^1$

②  $= D$ = unit ball in $\mathbb{R}^2$.

for f cts $f: D \longrightarrow D$

f must have a fixed pt.