Particular Point Top.

$X = \mathbb{R}$,

$\mathcal{T} = \{$ subsets that contain $\{0\}\}$.

$C = \{0\}$

Is C a cutset ?

Any open set contains $\{0\}$.

Say $(-1,1)$ is open but. $(-1,0) \cup (0,1)$ doesn't have $\{0\}$ isn't open

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(a) of the connected topological space X.

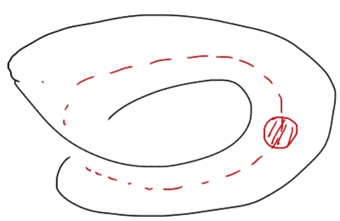
(a) $C = \{b\}$ and $X = \{a, b, c\}$ with topology $\{\varnothing, \{b\}, \{a, b\}, \{b, c\}, X\}$.

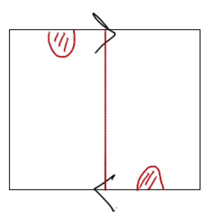
$X - \{b\} = \{a, c\} = U \cup V$ w/ U, V live here

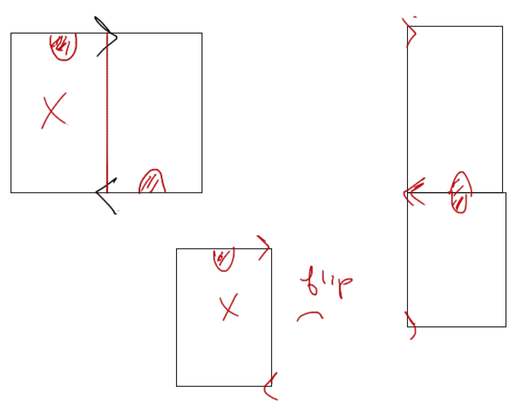
$\{x\}$ is a cutpoint in $X - \{x\}$ is disconnected

i.e., $\exists$ separation of $X - \{x\}$

$X - \{x\}$ = Disjoint union of open sets.







Path-Connectedness

Def'n: A path in a set X is a cts function
$$f:[0,1] \longrightarrow X.$$

(important: end points included.)

Given two points $a, b \in X$ a path between them is a

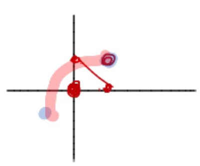
$f: I \to X$ s.t. $f(0)=a$, $f(1)=b$.

A set X is "path-connected" if there exists a path b/w any two points.

Ex. $\mathbb{R}^2$ is connected (std. top). It is also path-connected.
given $(3,4)$ & $(-7,-10)$. Define: $\boxed{f(t) = t(3,4) + (1-t)(-7,-10)}$

$f(0) = (-7,-10)$
$f(1) = (3,4)$

Ex. $\mathbb{R}^2 - \{0\}$ connected & path connected



Ex. $\mathbb{R}^2 - S^1$ not path connected (nor is it connected)

Ex. Topologist's sine curve

$S \subset \mathbb{R}^2$ w/ $S = G_1 \cup G_2$

$G_1 = \{(0,y) \mid -1 \le y \le 1\}$, $G_2 = \{(x, \sin(\frac{1}{x})) \mid x > 0\}$

S is not path connected. (Need infinite time to walk to any point on y-axis).

$f : [0,1] \longrightarrow$

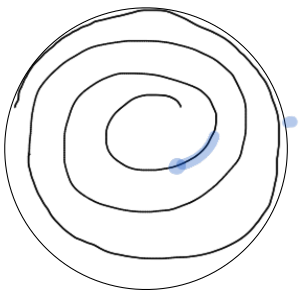
S is connected. Suppose U, V form a separation of S

Topologist's whirlpool

$S = \{ \frac{\theta}{\theta+1}, \theta \mid \theta \in \mathbb{R} \}$ polar coords

$r = \frac{\theta}{\theta+1}$

angle $= \theta$



U = unit circle

$W = S \cup U$ is connected, not path connected.