Math 366 - Topology
Final Exam Practice
December 3, 2021

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1. Determine if the following sets are connected and/or compact, in the given topologies on $\mathbb{R}$.

$$A = [1, 2) \cup \{3\} \ , \ C = (4, 5) \ , \ E = [6, 7] \ , \ Q = \mathbb{Q}$$

   (1.1) Discrete Topology
   (1.2) Standard Topology
   (1.3) Finite Complement Topology

2. Show that, in the standard topology on $\mathbb{R}$, $[1, 4)$ and $[2, 10]$ are not homeomorphic.

3. Show that, in the standard topology on $\mathbb{R}$, $[1, 4]$ and $[2, 10]$ are homeomorphic (construct a homeomorphism).

4. Show that, in the standard topology on $\mathbb{R}$, $(0, 1)$ and $\mathbb{R}$ are homeomorphic (construct a homeomorphism).

5. Prove that the continuous image of a compact set is compact.

6. Prove that the continuous image of a Hausdorff space is a Hausdorff space.

7. Prove that $\mathbb{R}^2 - \{0\}$ is not compact.

8. Let S^1 denote the circle, D the open disk, $\overline{D}$ the closed disk, and p a point in D. Describe in words and images the following product spaces and indicate whether the spaces are open, closed or neither in the subpspace topology on $\mathbb{R}^3$.
   (8.1) $S^1 \times D$.
   (8.2) $S^1 \times \overline{D}$.
   (8.3) $S^1 \times S^1$.
   (8.4) $\overline{D} \times \overline{D}$
   (8.5) $S^1 \times (D - \{p\})$.

9. Consider the unit square in the plane,

$$X = [0, 1] \times [0, 1].$$

   Describe five quotient maps on X that give surfaces. Give an argument why each resulting surfaces is <u>not</u> homeomorphic to any of the others.

10. Let $f : X \longrightarrow Y$ be continuous and assume that $A \subset X$. Prove that if $x \in \overline{A}$ then $f(x) \in \overline{f(A)}$.

11. Show that $\{\frac{1}{n}\}$ for $n \in \mathbb{N}$ is not closed in the standard topology on $\mathbb{R}$.

12. Show that $\{\frac{1}{n}\}$ for $n \in \mathbb{N}$ is not compact in the standard topology on $\mathbb{R}$. Use this result to answer the preceding question.