Friday – Week 2

Recall:

Basis:

1. "coverage" — $\forall x \in X \ \exists B_x \in \mathcal{B}$ s.t. $x \in B_x$.
2. $\forall x \in B_1 \cap B_2$ w/ $B_i \in \mathcal{B}$ $\exists B_3 \in \mathcal{B}$ s.t. $x \in B_3 \subset B_1 \cap B_2$

In a topology, $\mathcal{U}$ is open if $\mathcal{U}$ is a union of basis elts.
(or ϕ)

Union lemma
whole space
$X = \bigcup_{x \in X} B_x$

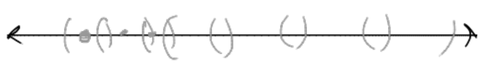
Ex. ($\mathbb{R}$, discrete)

A basis for this topology is all the singletons:
B/c any set is open in the discrete topology & any such set, is a union of its elements (its singletons).

$$\mathcal{U} = \bigcup_{x \in \mathcal{U}} \{x\}$$

Ex ($\mathbb{R}$, std)

A basis for this topology is all open intervals (a,b) w/ $a<b$ are real.



1. $\mathbb{R} = \bigcup_{x \in \mathbb{R}} (x-1, x+1)$ is open

2. $(-\infty, 7) = \bigcup_{n=0}^{\infty} (-n, 7) = (0,7) \cup (-1,7) \cup (-2,7) \cup \ldots$

$\mathbb{R}^2$

$d(p,q) = \sqrt{(p_1-q_1)^2 + (p_2-q_2)^2}$

Open ball centered @ x of radius ϵ (or ϵ-ball @ x)

$$B(x,\epsilon) = \{ y \in \mathbb{R}^2 \mid d(x,y) < \epsilon \} \quad (\epsilon \text{ is given/fixed})$$

Std Topology on $\mathbb{R}^2$

$\mathcal{B} = \{ \text{all } \epsilon\text{-balls in } \mathbb{R}^2 \}$

$= \{ B(x,\epsilon) \text{ s.t. } x \in \mathbb{R}^2,\ \epsilon > 0 \} \longleftarrow$ here x is variable, radius is variable.

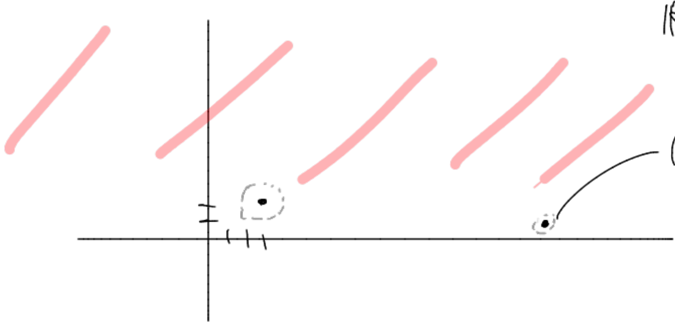
<u>Facts</u>:

Half-Planes are open.

$\{ (a,b) \in \mathbb{R}^2 \mid b > 0 \}$ is the upper half plane $= \Pi^+$.

This is open b/c every point of Π^+ lives in some basis elt, & this basis lives in Π^+.

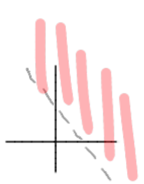
$\mathbb{R}^2$



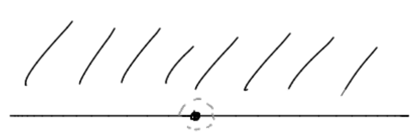
(m,n) w/ $n>0$

$(m,n) \in B((m,n), \frac{n}{2})$

<u>Ex</u>



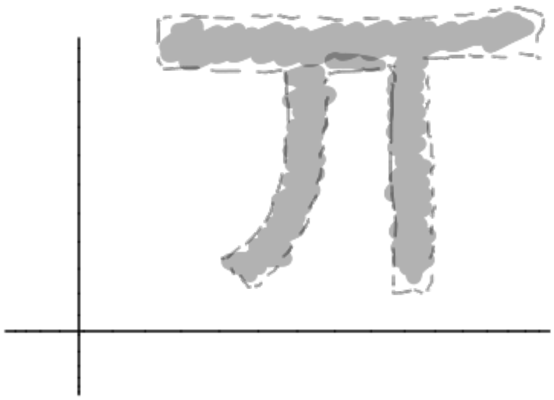
Non-Ex



$= \{ (a,b) \in \mathbb{R}^2 \mid b \geq 0 \} = X$

Not open b/c any ball that contains $(a,0)$ will <u>not</u> be contained in X.

Ex



open

Ex graph of $y = mx + b$ open? NO

Closed Sets

A subset C of a top. space X is closed if $X - C$ is open.

Ex:

$(\mathbb{R}, \text{std})$

1. $\{5,6\}$ is closed b/c its complement $\underbrace{(-\infty,5) \cup (5,6) \cup (6,\infty)}_{\text{open}}$

2. $\mathbb{Z} \subset \mathbb{R}$, $\mathbb{R} - \mathbb{Z} = \overset{\text{open}}{\bigcup_{n \in \mathbb{Z}} (n, n+1)} \Rightarrow \mathbb{Z}$ is closed

Ex. $(\mathbb{R}, \text{discrete})$

(a,b) is closed b/c $\mathbb{R} - (a,b)$ is a subset in the discrete top, $\therefore$ open

Every set is both open & closed

Ex. $\mathbb{R}^2$

is closed b/c its complement is a union of ½ planes