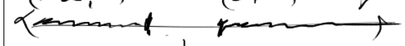
Finite Complement Topology on $\mathbb{R}$.

open sets: Def $\rightarrow$ the complement is finite

Since $\mathbb{R}$ is infinite $\Rightarrow$ open sets to be infinite

In general,
a closed set is defined to be
a set whose complement is open

$(-\infty, 2)$ $(5, \infty)$ $\rightarrow$ not closed



not open

$[2,5]$

$\mathbb{R} - \{5\}$ is open

$\{5,6,7\}$ is closed set

Q. What are closed sets in T_{FC} in $\mathbb{R}$?

closed $\Rightarrow$ complement is open

$\Rightarrow$ open = complement is finite

$\Rightarrow$ finite

What's a finite set? A set whose complement is $\mathbb{R}$ − finite set.

This is the def. of open in T_{FC}.

In standard Topology on $\mathbb{R}$

$(1,2) \cup (3,5)$ open

$[1,5) \overset{?}{=} \cup (\ , \ )$ $\Rightarrow$ NO, not open

$[1,4]$ = not open

implies

$\Rightarrow$ $[1,5)$ not closed

$\rightarrow$ {complement $(-\infty,1) \cup [5,\infty)$ not open.}

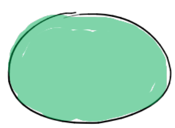
$\rightarrow$ complement $(-\infty, 1) \cup (4,\infty)$ is open

In $\mathbb{R}^2$, in standard topology

basis elts = open balls, open sets = unions of open balls

closed ball = $B(x,\epsilon) = \{ y \in \mathbb{R}^2 \mid d(x,y) \leq \epsilon \}$

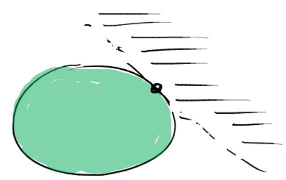
why is this closed?



1. Open half planes are open

why? you can cover half planes with open balls

2. complement of closed disk is union of half planes



vary the slope & pt.

Thm: (the properties of a topology for closed sets)

1. ϕ, X are closed.

2. $\bigcap_{\alpha} C_\alpha$ is closed (when each C_i is closed)

3. $\bigcup_{i=1}^{n} C_i$ is closed

Ex, $[-\frac{1}{n}+1, 2+\frac{1}{n}]$ closed sets for each $n \in \mathbb{Z}$.

$\bigcup_{n=1}^{\infty} [-\frac{1}{n}+1, 2+\frac{1}{n}]$ = set whose members live in some $[-\frac{1}{n}+1, 2+\frac{1}{n}]$ for a given n.

(1 nor 2 lives in this set)

why the finiteness restriction on unions

$= (1,2)$

we know the complement of closed is open.
what about the complement of open? $\Rightarrow$ it's closed!

Let U be open. Consider $X-U$. Show $X-U$ is closed.

$X-(X-U) = U$ is open. $\Rightarrow$ the complement of U is closed!